The Association
of
Engineering and Shipbuilding Draughtsmen.

The Powering of Ships.

(Some Present-Day Aspects).

By T. C. TOBIN, M.A. (Cantab.), M.Inst.N.A.

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THE GENERAL PROBLEM.

The question of the amount of power to be provided for any ship, so that a given speed may be maintained under specified conditions, is one of fundamental importance in Naval Architecture, and a short general survey of the present position may be of interest to all who are interested in the Design and Construction of Ships. Some of those who are responsible for the powering of ships are in a specially favoured position for making an estimate, inasmuch as they have at their disposal means of carrying out experiments to throw light on particular and difficult cases, but the greater number of designers have only their own immediate fund of collected data and the published researches of others to guide them ; it is mainly from the point of view of this latter class that the following brief survey has been written.

This problem, like a good many others, suffers frequently and severely from indefiniteness of statement. The owner who asks for his vessel to be driven at so many knots sea speed thinks, as a rule, that he has stated his requirements very precisely, ignoring the fact that the speed is sensitive to every change of wind and wave, and sea conditions may vary from the dead calm of a currentless ocean to weather of hurricane violence, and that a ship's performance when only half-loaded is no criterion of what she will do when fully laden.

It may be said at once that it is not a simple and definite engineering problem such as calculating the power necessary to raise a given weight through a certain height at a certain rate against gravity. From beginning to end, it is a process of balancing an expenditure of energy against the work involved in overcoming imperfectly ascertainable opposing forces, through the medium of structures and materials whose simultaneous efficient working is not always a certainty. The difficulty lies in the great number of factors involved ; energy leaking off at so many points between its generation in the prime mover and its final dissipation in eddies and waves. It must also be remembered that this energy passes through a strictly mechanical system, and any alteration in an

element of that system will affect the working of the whole, whether the alteration be in the engines, the form of the ship, or in the water and air through which she travels. Successful assessment requires due consideration to be given to all the links in the chain.

The elements of uncertainty in the case can be abundantly illustrated from the results of sister vessels, both on trial and in service. A recent writer quotes a case in which a group of vessels showed a much lower standard of performance than that of sister ships: investigation showed that those giving the poor result were all engined by one firm, those giving the good result by another firm which had had more experience in constructing that particular type of craft; here something analogous to the "personal" equation as applied to the practice of different firms enters the question. Another case is quoted,* in which four turbine steamers of identical dimensions showed a variation of 3½% in revolutions and 8½% in S.H.P. for given speed and displacement on trials in which there was practically no difference of weather conditions; here the assigned cause of the difference in performance was the state of the paint on the immersed surfaces. It has also been urged by one authority that there is a distinct gap in our knowledge of ship propulsion and resistance, that some factor or factors in the mechanical sequence between the engine and the hull resistance are unknown and in a particular case given where information was available from experiments with propeller and model in combination, there was a discrepancy of 9½% in the determination of the S.H.P., for which no explanation was forthcoming.†

These instances are sufficient to show that any investigation which throws additional light on the inter-relations of the links in the chain is of undoubted value, and also that the whole range of those relationships has not yet been fully explored.

It should be frankly recognised that it is not possible at present to calculate, *ab initio*, the power required to drive the ship; the whole process is one of continuous approximation based on the results of experience with actual vessels or on experiments with models. The history of the question during the last fifty years also makes it plain that whatever orderliness and precision there may be in our present-day knowledge of the subject, is due to the methods and results of research in the Experimental Tank as initiated by William Froude, and developed and extended by his successors.

This experimental method has enabled many problems to be investigated which would otherwise have remained matters of guess work; the effects of systematic variation of dimensions and form, the performances and characteristics of screw propellers; the effect

* "Shipbuilding and Shipping Record," 18/6/25.
† Sir J. Biles. Trans. I.N.A., 1924.

of rudders and hull appendages ; the effect of shallow water and of waves on the speed, are typical subjects on which a flood of light has been thrown.

In what follows, our knowledge of the subject will be dealt with from this experiential point of view, in so far as it concerns the driving of ships by means of screw propellers, as the considera-tion of other somewhat exotic or semi-obsolete types of propulsion would lead too far afield.

Leaving for the moment the question of the provision to be made to meet sea and weather conditions, and turning to the mechanical problem of providing power to drive a ship a specified speed in still water and calm air, it will be found that the factors group themselves about three main centres of interest, viz. :—

(*a*) The hull and its resistance,
(*b*) The propeller and its performance,
(*c*) The type of prime mover,

and we have to face the melancholy fact that before the power generated under (*c*) commences its real work under (*a*), more than half of it has, as a rule, been dissipated.

Before dealing in detail with the main sources of this loss, and the information gained thereon by modern methods of research, it may be well to consider some of the conditions under which model experiments must be made and interpreted so as to give reliable and intelligible results.

Since the dynamics of any moving body requires that the moving force be measured by the mass multiplied by the acceleration, the relation

$$F \propto M\,L\,T^{-2}$$

is always true where F, M, L, T, represent dimensionally the units of force, mass, length, and time respectively. In other words, since $M \propto L^3$ we get

$$F \propto L^3 \times L\,T^{-2} \propto L^3\,\frac{(L\,T^{-1})^2}{L} \propto L^3 \times \frac{V^2}{L}$$

where V is the velocity dimension.

Hence it may be concluded that the resistances of similar ships are in the ratio of the cubes of the linear dimensions when the speeds vary as the square roots of those dimensions. This is the well-known " Law of Comparison," as given by William Froude, and it is true where the dynamical conditions are similar, as in the case of wave-making, where we have

$$\text{wave resistance} \propto (\text{height})^2 \times (\text{breadth}).$$
$$\textit{i.e.}\quad ,, \quad ,, \quad \propto (\text{dimension})^3$$

It must not be supposed that the model method of investigation is confined to ship research ; it can and has been used under proper restrictions for determining the stresses in structures, the resistance of projectiles, and perhaps most noteworthy of all, it has been mainly responsible for the successful development of the modern aeroplane. The problems and results in the aeronautical work bear in many respects a close resemblance to those connected with ship models, and it has been found possible to verify in the wind channel some of the experiments of the tank. In each mode of experiment a fluid is being dealt with, in the one case water, in the other air, differing widely in their properties. The question at once arises, " Under what law or restrictions of comparison can an experiment conducted in one medium be compared with a similar experiment conducted in the other ? "

The answer to this is to be found in the consideration of those physical characteristics of a fluid which govern its motion. From the dynamical standpoint, fluids have two outstanding properties. The first and most obvious one is weight, or more particularly weight per unit volume, that is, specific gravity, or, if the term be preferred, " density," denoted very usually by the symbol ρ. This enters directly as a factor into all expressions for the kinetic energy of the fluid in motion, and for a given velocity is a measure of that energy. The other characteristic is that known as " viscosity." The meaning of this may be illustrated by considering the motion of a fluid taking place in a series of parallel planes in such a manner that if AB (Fig. 1) be one of the planes taken as a plane of reference, the velocity in any other plane CD is, say, $V_1 = a\, d_1$ and in any adjacent plane C^1D^1 it is $V_2 = a\, d_2$

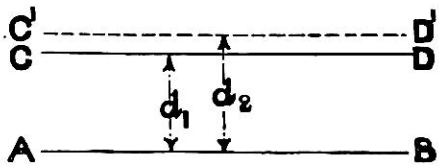


FIG. 1.

If the fluid is viscous, then the stratum CD will exert on the stratum C^1D^1 a retarding force whose amount per unit area of the plane CD will be some constant times the variation of the velocity per unit distance from AB ; that is, if this constant be denoted by the symbol μ, the retarding force per unit area between the planes will be

$$\mu \times \frac{V_2 - V_1}{d_2 - d_1} \quad i.e. \quad \mu\, a$$

The quantity μ is termed "the coefficient of viscosity," and obviously measures the frictional or tangential force exerted between adjacent layers of the fluid when in motion. The effect of this interplanar force is to modify the velocity of the adjacent fluid, and, as force is always measured by mass multiplied by change of velocity, we see that the change of velocity is measured by the force divided by the mass; in other words, by $\frac{\mu}{\rho}$. Hence, in any comparison of the motion of one fluid with another, or of bodies moving in fluids, similarity of motion depends on a consideration of this quantity $\frac{\mu}{\rho}$, to which a special symbol ν has been allotted, termed by Clerk Maxwell the "Kinematic Coefficient of Viscosity."

When comparing model results with those of the full-sized ship, due allowance must, therefore, be made for the kinematic viscosity effect, and this necessitates a special law of comparison which was first pointed out by Osborne Reynolds, and formulated later, from hydrodynamical considerations, by Lord Rayleigh. When William Froude proposed the model method, his object was to run the model at such a speed that the wave pattern created by the model would be exactly similar geometrically to that created by the ship; this led him to his law of comparison for corresponding speeds, and as the wave energy was practically independent of the viscosity, his law applies, strictly speaking, to this wave-making resistance. His conception of a frictional resistance, apart from the wave-making, practically recognised the portion of the resistance due to what we are now referring to as "viscosity," and as he had no law of comparison for it, he resorted to separate experiment to determine its value, subtracting the amount so determined from the total resistance of the model, in order to determine the wave-making portion. This procedure was fundamentally sound, although he was apparently unaware of the law which should govern the frictional comparison. This law, as indicated, has been formulated by Rayleigh, who has shown that where the forces on a body moving in a fluid arise solely from the viscosity, then in comparing the motion of two similar bodies of different scale moving in different fluids, the ratio of the resistances will be given by :—

$$\frac{R_1}{R_2} = \frac{\rho_1 l_1^2 v_1^2}{\rho_2 l_2^2 v_2^2}$$

provided that $$\frac{l_1 v_1}{\nu_1} = \frac{l_2 v_2}{\nu_2}$$

where ρ, v, l, ν, refer respectively to density, speed, linear dimension, and kinematic viscosity.

The difficulties in the application of this law are very much greater than in the case of the Froude law, in which the speeds of the models are very much lower than the speeds of the full-sized ships. If the fluids are the same, it will be seen at once that for the Rayleigh Law we must have $v_1 l_1 = v_2 l_2$, that is to say, the smaller the model, the greater must be the speed, or if the ship were 400 ft. long, travelling at 10 knots, then a 16-ft. model would have to run at a speed of 250 knots. Or, from the resistance point of view, the resistance of the model would have to be exactly equal to that of the ship, because if $\rho_1 = \rho_2$ and $\nu_1 = \nu_2$ we get by the law :—

$$\frac{R_1}{R_2} = \frac{l_1^2 v_1^2}{l_2^2 v_2^2} = 1 \qquad \text{Since } l_1 v_1 = l_2 v_2$$

It should, however, be observed that a complete comparison between the ship and the model could be obtained, including both frictional and wave-making resistance, provided we could find a fluid of the same density as water with a sufficiently low viscosity. To secure this direct comparison, it would be necessary to combine the two sets of conditions belonging to the Froude and Rayleigh Laws, viz. :—

$$\frac{g\,l_1}{v_1^2} = \frac{g\,l_2}{v_2^2} \quad \text{and} \quad \frac{l_1 v_1}{\nu_1} = \frac{l_2 v_2}{\nu_2}$$

These will be simultaneously satisfied if

$$\frac{v_1^3}{v_2^3} = \frac{l_1^{3/2}}{l_2^{3/2}} = \frac{\nu_1}{\nu_2}$$

Hence, if it were possible to obtain a fluid whose viscosity was $\frac{1}{125}$th that of water, then the total resistance of a 400 ft. ship travelling at 10 knots speed would be ascertained by a direct comparison with the results from a 16-ft. model towed at the rate of two knots. It is the impracticability of obtaining a fluid of this nature which makes necessary the separation of the wave-making resistance from the frictional.*

Passing to the case of the application of the principle of comparison to model propellers, it is necessary to consider the effect of size, revolutions, speed of advance, density, viscosity, and elasticity of the fluid ; and where the experiments are in water, the effect of gravity if the immersion is not sufficient to eliminate it. It is found that four sets of conditions are involved, viz. :—

$$(1)\quad \frac{n_1 d_1}{v_1} = \frac{n_2 d_2}{v_2} \qquad (3)\quad \frac{g\,d_1}{v_1^2} = \frac{g\,d_2}{v_2^2}$$

$$(2)\quad \frac{v_1 d_1}{\nu_1} = \frac{v_2 d_2}{\nu_2} \qquad (4)\quad \frac{v_1}{V_1} = \frac{v_2}{V_2}$$

* Cf. "Principle of Similitude, etc." T. E. Stanton, British Association, 1916

Where n, d, v, ν, V, refer respectively to the number of revolutions, dimensions, velocity of advance, kinematic viscosity, and the velocity of sound in the fluid. Now, it should be observed that if only one fluid be used $\nu_1=\nu_2$ and $V_1=V_2$; therefore we must have $v_1=v_2$ and, consequently, $d_1=d_2$; the implication being that in this case small scale experiments would yield no intelligible result. In practice, however, it is found that all these conditions are not required, and that for experiments in water the two conditions—

$$\frac{v_1}{V_1} = \frac{v_2}{V_2} \quad \text{and} \quad \frac{v_1 d_1}{\nu_1} = \frac{v_2 d_2}{\nu_2}$$

can be ignored, which is the same as saying that the speeds involved are small as compared with the velocity of sound in water and the viscosity effect is negligible as compared with the turbulent one. We are then left with two conditions only to be fulfilled, viz. :—

$$\frac{n_1 d_1}{v_1} = \frac{n_2 d_2}{v_2} \quad \text{and} \quad \frac{d_1}{v_1^2} = \frac{d_2}{v_2^2}$$

which can be satisfied by making the " slips " the same and using the Froude Law of comparison for the speeds. The ratio of the thrusts will then be given by

$$\frac{T_1}{T_2} = \frac{\rho_1 d_1^3}{\rho_2 d_2^3} = \frac{\rho_1 d_1^2 v_1^2}{\rho_2 d_2^2 v_2^2}$$

Turning now to the particular problems connected with the Estimation of Power, it is proposed, in the first place, to consider those connected with the Hull and its resistance. These in the course of time have grouped themselves round three sub-headings—Skin Friction ; Eddymaking ; and Form Resistance, which it is proposed to review in this order.

(a) THE HULL AND ITS RESISTANCE.

Skin Friction.

As used by the early investigators of Ship Resistance, this term was employed to denote that portion of it attributable to the action of the water on the immersed surface, as distinct from the bodily resistance which caused the formation of waves.

The term, in a broad sense, is appropriate enough, and describes one part of the fluid resistance which is of undeniable importance, as it certainly comprises from 50% to 80% of the total amount for the great majority of ships in existence, and is consequently, from the Owners' point of view, the largest factor affecting the fuel bill.

The familiar Admiralty Formula is, by implication, based on the assumption that the resistance is almost wholly frictional ; for the

quantity — (Displacement)$^{\frac{2}{3}}$ — from a dimensional point of view, represents a surface, and the wetted surface of the vessel is roughly proportional to it. It is probably on account of this that the formula has persisted so long as a means of approximating to the value for the total resistance or the Horse Power.

Any estimate of the frictional resistance must be based on experiment, and the history of the experimental work for this purpose is now a long one, dating back, as it does, to the experiments of Colonel Beaufoy in 1793 ; but it was not until 1872-4 that William Froude produced an empirical formula, which, with R. E. Froude's extension in 1888, has until recently been the accepted method of calculation. During the last twenty years, however, largely due to aeronautical developments, interest in the subject has greatly increased, and a large volume of experimental work has been done, and distinct advances made in the theory of the attendant fluid motion. The principal experimenters in water since Beaufoy and the Froudes, have been Gebers at Vienna and Dresden, Baker at the National Tank, Kempf at Hamburg, and McEntee & Taylor at Washington, whilst Zahm and others have conducted experiments in a wind channel. Until recently, all the experiments have been made with "planks," some fitted with cut-waters at each end, some wholly submerged, some only partly so ; some were planks in the ordinary sense, and others were triangular in cross section. It may be said at once that these experiments on the whole were very carefully conducted, and the results obtained are reliable within the range of observation ; but there were physical limitations in connection with the work, and the planks that could be handled were short as compared with ships, and in this lies the crux of the situation ; for on what grounds is it reasonable to suppose that the resistance of a 50 ft. plank can be used as a measure for the resistance of a ship, say, 400 ft. long of a totally different shape ?

Froude expressed the results of his investigations in the form :—

$$\text{Resistance} = f\ A\ V^n$$

Where A is the immersed surface and V the speed, f and n being numbers varying with the condition of the surface, etc. He assigned values of f and n for the whole plank and estimated their values for the last foot of each plank, and his method of extension to long surfaces was to assume that each foot of length beyond the first 50 ft. was subject to the same resistance as the last foot of the 50-ft. plank.

The modern and hydrodynamical way of recording the phenomena concerned was initiated by Lord Rayleigh and Osborne Reynolds. Rayleigh pointed out the importance of the quantity which was termed the "Kinematical Viscosity," in comparing the motions of different bodies in the same or different fluids ; this

quantity (ν), as previously explained, is the ratio of the ordinary statical viscosity to the density of the fluid, and Rayleigh found that the law of resistance comparison should follow the form :—

$$R = \tfrac{1}{2}\,\rho\,V^2\,F\left(\frac{V\,L}{\nu}\right)$$

Where ρ is the density, and $F\left(\frac{V\,L}{\nu}\right)$ denotes some function of $\frac{V\,L}{\nu}$, to be determined.

This means that the quantity $\frac{R}{V^2}$ should be the same for similar bodies of whatever length and surface at the same value of $\frac{V\,L}{\nu}$

Using this principle as a basis of comparison, it is found that the experimental work hitherto carried out falls very far short of providing values of $\frac{V\,L}{\nu}$ comparable with those required for actual ships. Froude and Kempf worked with higher values than the other experimenters, Froude's highest value being $54{\cdot}5 \times 10^6$, and that of Kempf $213{\cdot}4 \times 10^6$, whilst the value required for a 400 ft. ship at 10 knots speed is 517×10^6*.

Whilst, however, it is found that the experiments give good agreement amongst themselves when compared on this modern basis, the actual resistances of ship forms are apparently higher than those estimated from the plank experiments, and the problem of the mode of extension still remains.

Froude's extension was based on the results he obtained by towing H.M.S. "Greyhound," whose length was 172·5 ft., but as the roughness of the surface of this ship was an uncertain quantity, the law could not be said to be established by the result, and generally it may be said that the correspondence between model and ship results cannot be held to verify the frictional law, as other factors may mask the variations ; empirically, it is only safe to assess values up to the limits of the experimental observations.

In 1915, the N.P.L. Tank Authorities proposed to effect the extension by means of a formula :—

$$R = k_1\,A\,V^2\left(\frac{V\,L}{\nu}\right)^{-\cdot 138}$$

and Baker† has given a straight line diagram showing this in a convenient form for calculation.

* Skin Friction Committee's Report. Trans. I.N.A., 1925.

† "Ship Form and Resistance." G. S. Baker.

Gebers has proposed a very similar form :—

$$R = k_2 A V^2 \left(\frac{V L}{\nu}\right)^{-\cdot 125}$$

It will be observed that each of these formulae, like Froude's, expresses the resistance in terms of a single power of V, and, whilst for a limited range of speed a formula of this type can be made to represent the experimental results with tolerable accuracy, this limit is soon passed. Several investigators, such as Blasius, Gümbel, Shigemitsu, and Kempf, have therefore suggested that the frictional resistance should be regarded as made up of two portions :—

(*a*) That due to the thin boundary layer, one of whose sides is in contact with the moving surface and moves with it, and the other side is in contact with a turbulent wake. In this thin layer the fluid motion would be of a viscous nature.

(*b*) That due to the waste in the turbulent wake outside the thin boundary layer.

These divisions may be broadly termed the viscosity resistance and the inertia resistance, and any formula obtained on this basis would necessarily be of the form :—

$$R = AV^2 (f_1 + f_2)$$

Where f_1 and f_2 are functions of $\frac{V L}{\nu}$ and, if determined, would enable the relative importance of the two types of resistance to be estimated ; the extension of the formula however to lengths and speeds beyond the experimental range would still be open to the same objections as in the case of the Froude formula.

Kempf's work* is illustrative of this binary type of expression. He experimented with long cylindrical tubes in the Hamburg Tank, the maximum length of tube employed being about 202·5 ft., its external diameter being 13·78 inches. He found generally that at a constant speed the resistance increases directly in proportion to the increase in the surface of the pipe, and that the unit of area had a specific resistance independent of its position in the length ; this being ascertained by inserting a short test length at various positions along the pipe, and verified by experimenting on paraffin models of barges. He also found that there was an entry zone at the fore end extending for 10 to 15 ft. over which the constancy of specific resistance did not hold good. Abaft this entry zone, he found a definite relation between the two types of resistance

* " Skin Friction Experiments and Results." Dr. G. Kempf, Delft, 1924.

already mentioned. If R_1 represents the inertia term, and R_2 the viscosity term, then by assuming—

$$R_1 = f \rho V^2 \text{ and } R_2 = f \mu \frac{du}{dx} = f \mu \frac{v}{q}$$

where v = absolute speed ; ρ = density ; μ = viscosity ; u = speed at distance x from the tube ; and q a parameter depending on the roughness, he found an expression for the total resistance in the form

$$R = f \rho \lambda \left(v^2 + \frac{\mu}{\rho q} v\right) = f \rho \lambda (v^2 + a\, v)$$

and that this held good for both plank and pipe experiments ; λ being a constant depending on the quality of the surface.

It will be seen that for a particularly rough surface, the term av would disappear, and the resistance would follow the quadratic law only, which is reasonable ; and, further, it emphasises the inadvisability of using a single power formula, as it involves the assumption that the viscosity resistance increases with the speed, which is not probable. Kempf varied his experiment by using a pipe only 1·378 in. diameter, and found that the value of λ was increased by some 10%. which showed that for equal roughness the resistance was increased as the radius of curvature of the surface perpendicular to the direction of motion was decreased ; hence the form of the body had an appreciable effect on the frictional resistance. In English units, the final expression for frictional resistance and horse power was given by Kempf as—

$$R = \cdot 0062 \text{ AV } (V + \cdot 64)$$
$$E.H.P_f = \cdot 0000190 \text{ AV}^2 (V + \cdot 64) = 19 \text{ AV}^2 (V + \cdot 64) \times 10^{-6}$$

Where V = speed in knots ; R = resistance in lbs. ; A = wetted area in square feet.

The formulae now available for estimating skin friction lead to a variety of results, a fact which has been pointed out by the "Skin Friction Committee" in their Report*, where it is shown that for a ship 400 ft. long with a wetted surface of 30,000 square feet, and travelling at 25 knots, there would be obtained for the frictional resistance :—

(1) 94,600 lbs. by W. Froude's method for ship ; (2) 94,400 lbs. by R. E. Froude for plated ship ; (3) 85,100 lbs. by the N.P.L. Tank for a smooth, and 93,600 lbs. for a plated surface ; (4) 80,000 lbs. by Gebers for smooth surface ; (5) 128,200 lbs. by Kempf for a painted iron surface.

The difference in frictional resistance which Kempf found between tubes of the same length but different diameter opens up the

* Skin Friction Committee's Report. Trans. I.N.A., 1925.

general question of the effect of "Form" on frictional resistance. A ship-shaped body differs radically from a plank in this respect, and, on the other hand, it must not be forgotten that planks themselves have shape. In fact, experiment has shown that the depth and thickness of a plank have an important bearing on its frictional resistance; the presence of the edges modifies the fluid action as well as the bluntness or otherwise of the ends.

Experiments have been and are being carried out to investigate this question generally as it applies to ships.* Broadly speaking, it has been found that the Rayleigh Law of Comparison holds good, but there are minor departures due to depth, beam, and body effect.

In attempting to run models at very low speeds, so as to eliminate wave-making resistance, various difficulties are encountered. Perring* ran a series of seven models of rectangular section with circular bilges, each having a curve of areas identical with that of a mathematical torpedo-like form, for which he had experimentally determined the stream line motion, the same girth being kept in each case; he found an excess of resistance above the Froude value of 10% to 15% which was directly attributable to the "Form" effect. Two of these models, however, showed a remarkable phenomenon at very low speeds, in that there appeared in each case the possibility of either of two types of motion being set up round the model, giving a distinctly different resistance according as the motion was of one type or the other. Generally speaking, very unstable results were found for values of $\frac{V L}{\nu}$ less than 4×10^6, which corresponds approximately to 10 knots for a 400 ft. ship.

A further series of models run with varying ratio of beam to draft showed that as this ratio decreased the excess above the Froude curves decreased from 10% to about 5%, but on reducing the ratio to 1·443 the excess went up again to 13%. The explanation which is suggested for this unexpected variation is that the fluid motion round the Model changes character from three-dimensional to two-dimensional when a certain critical point in the ratio is passed. It is, therefore, evidently possible at certain speeds and with certain shapes to find a serious departure from the Froude extension formula, and it is only to be expected that, in view of this, the subject of "Form" effect has now a definite place in the scheme of researches to be carried out at the William Froude Tank of the National Physical Laboratory.

Where then, it may reasonably be asked, does the designer stand amid all these discrepant figures? As close predictions of horse

* "Form Effects and Form Resistance." W. G. Perring. Trans. I.N.A., 1925.

power have been and continue to be made, do the differences here indicated have a real existence ?

Possibly the best way of answering this question is to suppose that these extension formulae do underestimate the frictional resistance by some stated amount, and then find what effect this given error will have on the final prediction of the ship resistance. This is not difficult, and a general case has been worked out by Perring,* who gives the results in the form of a diagram showing the percentage error in ship prediction for a given percentage error in the estimated friction due to treating the ship as a plank. Comparing the ship with a model $\frac{1}{25}$th full size, the present writer has found that the whole diagram may be conveniently summed up in the comparatively simple formula :—

$$e = \cdot 715 \times \frac{F}{R} \times x$$

Where e = % error in the ship prediction ; x = % error due to plank basis for estimate ; $\frac{F}{R}$ = ratio of frictional to total resistance.

For example, if the frictional resistance is underestimated by 12%, then if $\frac{F}{R} = \cdot 7$, the error in the ship prediction would be 6%, and so on. It would thus appear that there is a real possibility of error in the estimate for the ship, and that it is not negligible ; but until further light is thrown on the question experimentally, it must be allowed for in the general coefficient of performance.

Eddymaking.

The portion of the ship's resistance which it is now usual to assign to eddymaking occupies an intermediary position between the frictional and the wave-making resistances. It is partly produced by the fluid friction, and partly by abrupt or quasi-abrupt changes of form. Experiment also shows that it increases with the speed. It has been customary to treat it as part of the residuary resistance, which implies that its amount is taken to vary as the square of the velocity. It cannot be made the subject of direct calculation, and the best that can be done is to avoid, as far as possible, the conditions which favour its production. Experiments, both on ship models and balloon-shaped models have shown that in almost every case the eddies commence to form when the slope of the surface exceeded some 16° to 18° from the line of the form

* W. G. Perring. *loc. cit supra.* Skin Friction Committee's Report. Trans. I.N.A., 1925.

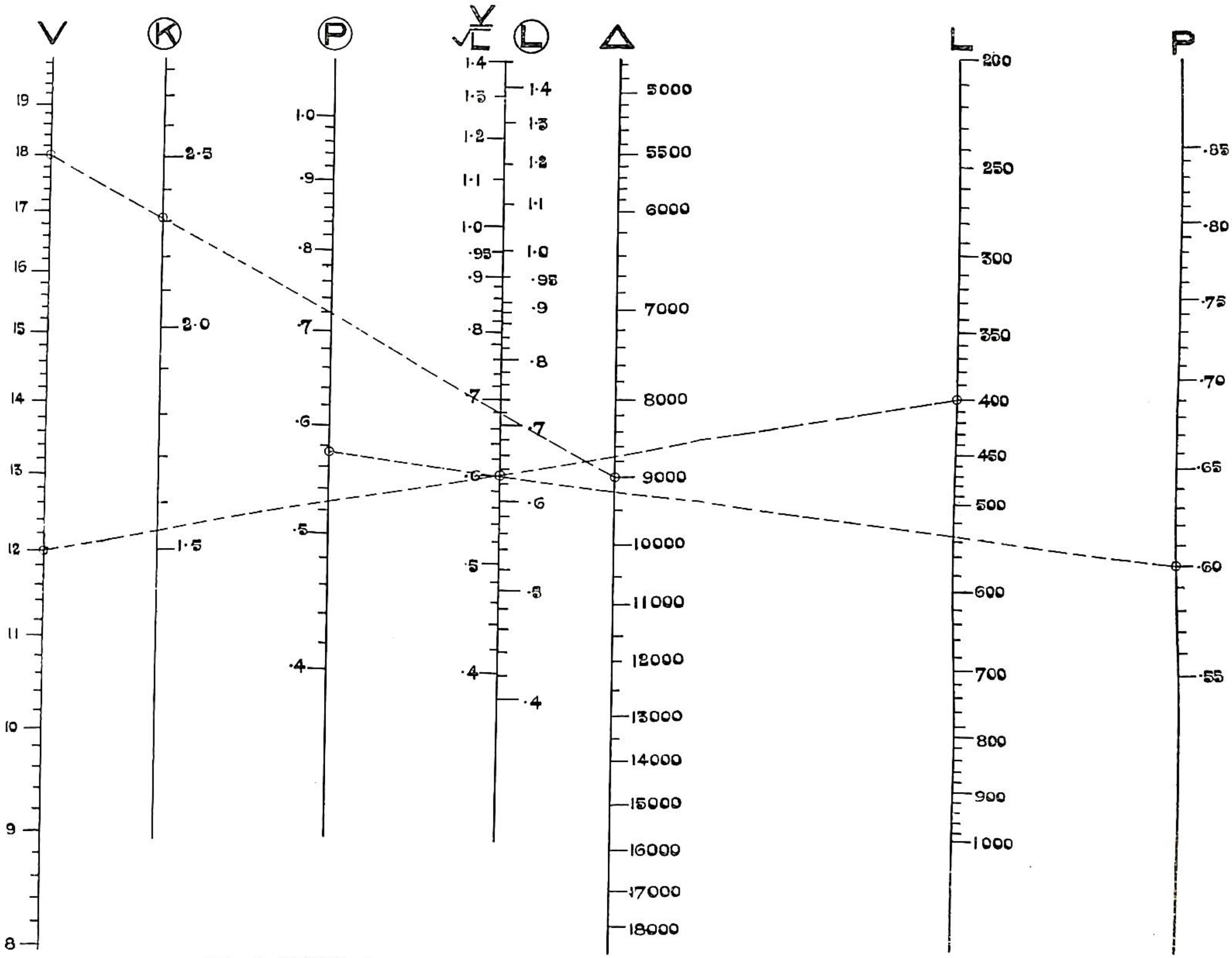
axis.* If possible, therefore, the slope of the run aft should not exceed this angular limit, and as much as possible should be done to minimise the eddymaking effect of bilge keels, shaft brackets, etc. This is as far as modern research has taken the question, and the loss of power is masked in the general turbulent and wave-making loss.

Form Resistance.

It has already been pointed out that the form has an effect on the frictional resistance; but this is a comparatively small portion of the total form effect, the main part of which is shown in the wave-making characteristics of the vessel. The history of our knowledge of the subject dates, of course, from the classical experiments of William Froude, and it has only been by following out the lines of investigation which he commenced that any advance has been made towards the solution of a very complicated problem. The experimental work of the last twenty years carried out in America, England, and elsewhere has, however, done much to clear the air and bring a certain amount of coherence into our ideas, opening up avenues of approach to an exact appreciation of what constitutes good and bad form. The old idea of general eye-sweetness has given place to an analysis of characteristics; hap-hazard and semi-casual experimenting has been superseded by systematic investigation along definite lines, and the number of Experimental Tanks in which the work can be carried on is now seventeen.

Generally speaking, the results obtained in the various tanks confirm and supplement one another, and the resistance determinations of the models are stated to be accurate within $1\frac{1}{2}\%$; but it should also be remembered that each tank has its own "personal" error, and when comparing the results of different tanks, this should be allowed for; the difference varying up to some 3% according to the particular tanks which are being compared. The designer, who is dependent on published researches, is also faced with the somewhat inconvenient fact that the notations adopted by the various establishments in recording their results differ, and care is needed in passing from one system to another in dealing with a particular estimate of power. In what follows, the Froude constant system of notation is used. The Chart shown in Fig. 2 provides a simple means of determining the corresponding values of the various speed constants, and Fig. 3 gives a similar means for ascertaining the (M) values on this system. The American method of separating the frictional resistance from the wave-making, and measuring the latter in terms of its ratio to the displacement, is not so con-

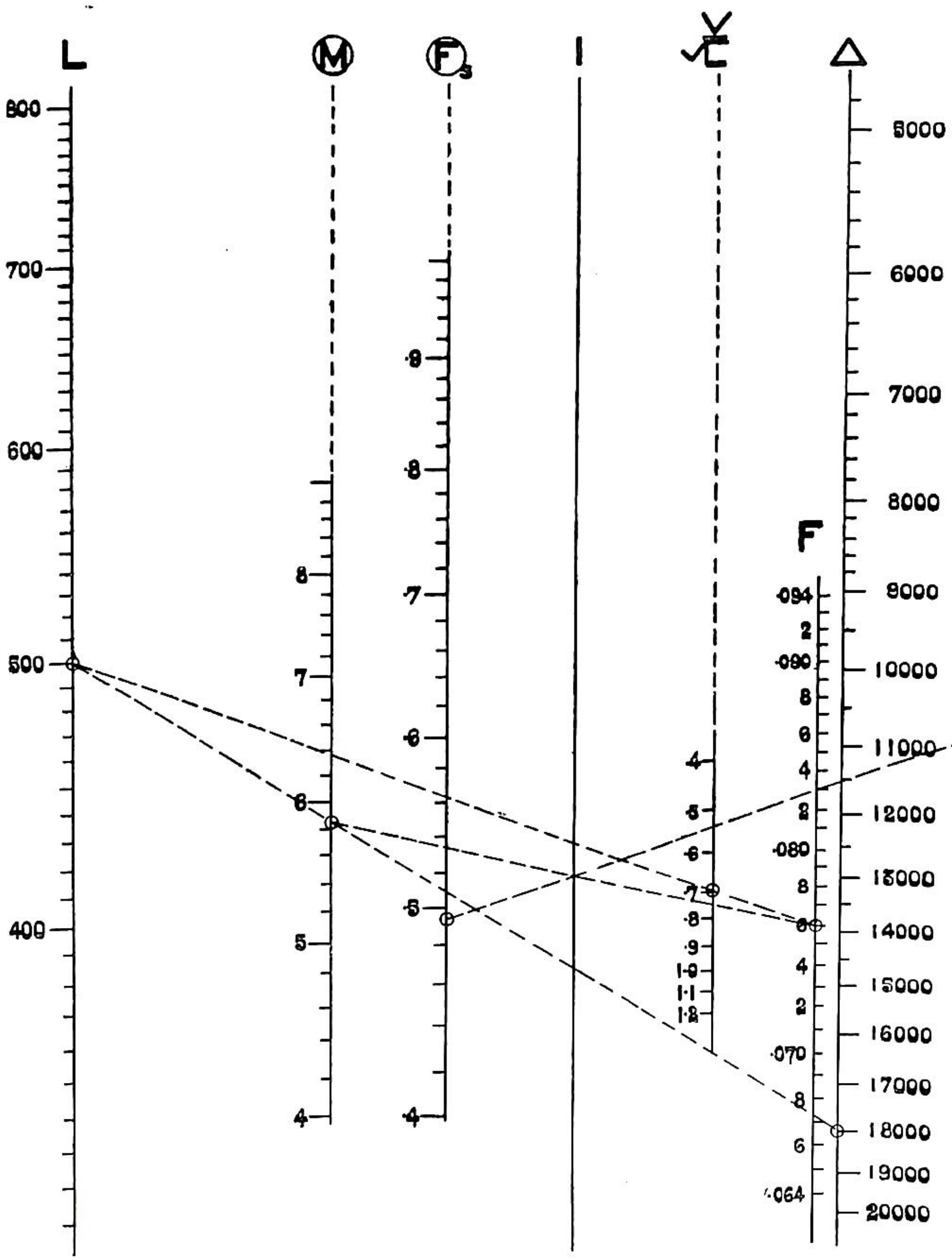
* *Vid.* "Ship Form and Resistance and Propulsion." G. S. Baker.



THE DOTTED LINES SHOW THE LINEAR INTER-RELATION OF THE SCALE POINTS INDICATED BY SMALL CIRCLES.

FIG. 2.

See Page 42.



THE DOTTED LINES SHOW THE LINEAR INTER-RELATION OF THE SCALE POINTS INDICATE BY SMALL CIRCLES.

FIG. 3.

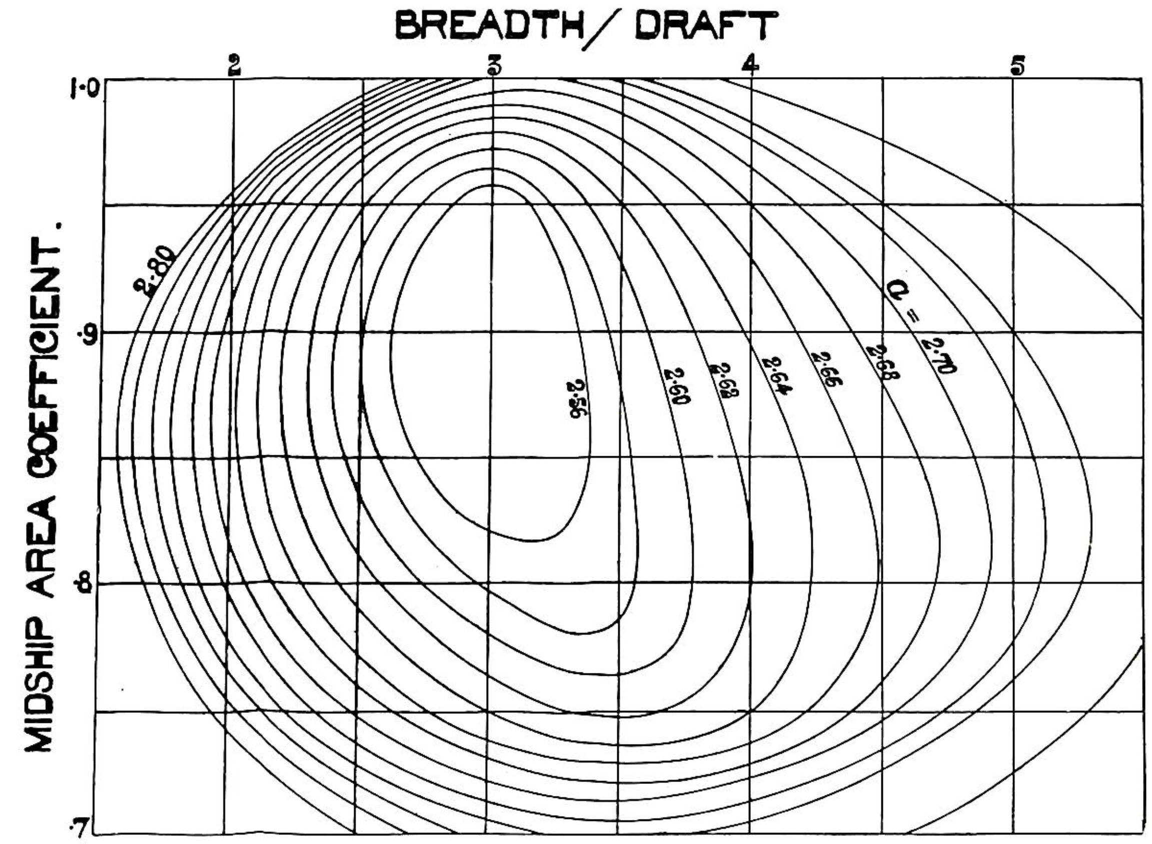
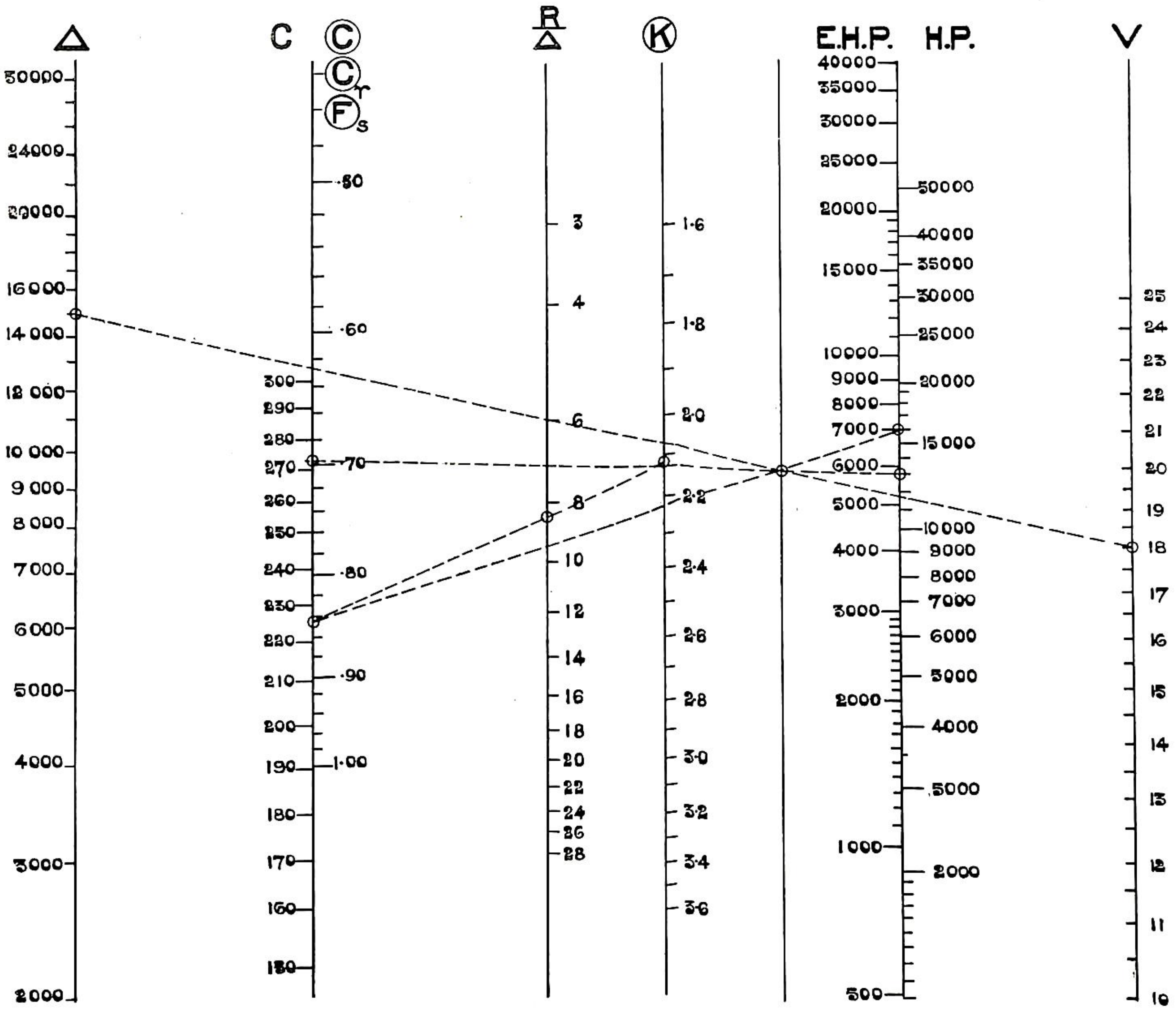


DIAGRAM FOR VALUES OF "a" FOR USE IN CONNECTION WITH FIG. 3.

FIG. 3A.

See Page 43.



THE DOTTED LINES SHOW THE LINEAR INTER-RELATION OF THE SCALE POINTS INDICATED BY SMALL CIRCLES.

FIG. 4.

See Page 44.

venient for showing the variations of the whole resistance of the model, although it can be easily linked up with the Froude constant system by means of Fig. 4.

The energy lost through wave-making is dissipated mainly in three distinct ways; first, by the formation of the waves whose crests are transverse to the line of the ship's motion; secondly, by the diverging waves from the bow and the stern; and, thirdly, by the interference of the bow and stern systems. The amount and the character of the loss in each of these three divisions depends on the speed of the vessel and the form of its under-water portion. The main features of the form which determine whether it is more or less resistful are now becoming much more clearly defined as the result of experimental work. Broadly speaking, for a vessel of given length, its wave-making properties are found to depend chiefly on :—

(*a*) The amount and distribution of the cross-sectional area fore and aft;

(*b*) The shape of the load waterline;

(*c*) The beam.

If the distribution referred to in (*a*) is shown in the form of a curve, this may have one maximum ordinate, or the ordinate may have a maximum constant value for a portion of the length; this distinction divides vessels into two classes—those without parallel middle body, and those with it. R. E. Froude and D. W. Taylor experimented extensively with ships having no parallel middle, and with prismatic coefficients ranging from about ·50 to about ·64, and the general results of their investigations showed that the variation of performances turned on the hollowness or otherwise of the area curve at the ends, and the combination of a fine or full load waterline with particular curves of areas, and also that the effects obtained by these combinations varied over different ranges of speed.

Cross-Channel Passenger Boats illustrate this type of vessel, which is confined to those having a low value for the prismatic coefficient. As this latter is increased, the necessity of introducing parallel middle body arises if, in general, shapely ends are to be maintained, and this introduction may be carried out in different ways. Froude, D. W. Taylor, the Experimenters at the National Physical Laboratory, and others, have made systematic researches on the effect of varying the form in this manner, and it can be taken as established that the performance depends on the length of the parallel middle, its relative position fore and aft, and the relative finenesses of the entrance and the run.

The shape of the midship sectional area can be varied over wide limits without materially affecting the resistance, but this area in

conjunction with the beam has an important bearing on the performance, and it has been found that the ratio $\frac{B}{\sqrt{A}}$ where B is the beam and A the midship area, is a determining quantity in the comparison of different forms.

So far, therefore, as form is concerned in producing resistance, its effects may be grouped under eight headings when referred to a given length of ship :—

(1) The total value of the prismatic coefficient, ... P

(2) The ratio of the "entrance" prismatic to the "run" prismatic, ... $\frac{P_e}{P_r}$

(3) The beam width, ... B

(4) The ratio of the beam to the square root of the midship area, ... $\frac{B}{\sqrt{A}}$

(5) The length of parallel middle body, ... M_l

(6) The ratio of the length of entrance to the length of run, ... $\frac{L_e}{L_r}$

(7) The shape and fullness of the load waterline, forward, ...

(8) The shape and fullness of the load waterline, aft

Any given Model may be varied in any of these ways or in any combination of them, or numerically—there are 185 modes of variation with these variables, and for each mode the amount may be anything we choose to make it. With some of the variables it is also possible to make the variations in different ways. The mere statement of this fact is sufficient to render obvious the difficulty, if not the impossibility, attending any attempt to express the resistance of a ship in a comprehensive formula.

The effect of length on the resistance shows itself most clearly in the wave-making, and particularly in the variations due to interference. It has been found that this is closely associated with a certain quantity (P) which is the ratio of the speed of the ship to the speed of a wave whose length is given by the product P × L, *i.e.*,

$$\text{(P)} = \cdot 746 \frac{V}{\sqrt{P \times L}}$$

and by plotting the (C) values obtained from experiment on a base of (P), it will be seen that the increases and diminutions of these values due to interference effects occur with tolerable consistency at the same definite values of (P).

Although the complexity of the problem is apparent, it has not deterred various investigators from attempting to produce a general formula for ship resistance. The oldest, and perhaps the most persistent, is the well-known Admiralty Formula, which in terms of Ⓒ becomes :—

$$Ⓒ = 427{\cdot}1 \times \frac{E\,H\,P}{\triangle^{2/3} \times V^3}$$

the companion formula involving the area of the midship section having disappeared through sheer lack of utility.

As the Admiralty Formula does not allow for the performance of the ship being separated from that of the propelling apparatus, the desire to know the resistance of the ship as a separate factor gave rise to a series of formulae designed for this purpose. In these, the "skin" portion was usually treated on the lines of Froude, but the residuary part assumed a variety of forms. Middendorf, for example, gave

$$R = \frac{{\cdot}039 \times A \times M \times V^{2{\cdot}5}}{\left[1 + C\left(\frac{L}{B}\right)^2\right]^{\frac{1}{2}}}$$

Where $A = f\left(\frac{L}{V^2}\right)$

M = Immersed Midship Area.

$C = 4{\cdot}7 - {\cdot}32\ \frac{L}{B}$

This gives only a fair approximation to model results over a limited range of size and speed, and, further, does not conform to the Law of Comparison. Admiral Fournier produced a most complicated one-term formula giving the total resistance and depending entirely on the coefficient of surface friction ; this agreed very closely with the results obtained from the models Fournier had used in making it, but broke down completely when applied to other cases ; it also did not conform to the Law of Comparison. D. W. Taylor has given for the residuary resistance :—

$$R_{lbs.} = 12{\cdot}5\, b \times \triangle \times \frac{V^4}{L^2}$$

Where b = Block coefficient.
$\triangle$ = Displacement in tons.

He limits this in its application to the cases where $\frac{V^2}{L} < 1{\cdot}2$.

As compared with other formulae, it does give quite good results for all classes and sizes of ships, and it does conform to the Law of Comparison. Johns* found that this formula gave correspondence

* Approximate Formulae for Ship Resistance." A. W. Johns, Trans. I.N.A., 1907.

with model results at particular speeds for different ranges of b value, the error being $7\frac{1}{2}\%$ to 10% greater than experimental values for the lower speeds, and slightly less than the observed figures for higher speeds. There was still, however, a variation from experiment of 10% to 15%, or even more, in extreme cases.

Johns* has proposed a formula of the form :—

$$\text{Resid. H P} = B \times \triangle^{7/6} = A \left(\frac{\triangle}{14{,}000}\right)^{7/6}$$

for speeds up to $\dfrac{V^2}{L} = 1{\cdot}3$ where A and B are coefficients determined by a series of curves plotted on a base of prismatic coefficient ; $\triangle$ being the displacement in tons. In this case again, the ship is merely defined in terms of displacement and prismatic coefficient, and there can be an infinite number of forms with these characteristics, varying considerably in resistance.

Tutin† starting from Taylor's formula as a base has suggested that the residuary resistance may be represented by

$$R = k \frac{B^2}{L^2} \sqrt{A}\, V^4$$

where A is the midship area ; and k a form coefficient.

This, it will be seen, endeavours to take account of both dimensions and form ; and it will also conform to the Law of Comparison. It will, however, have the same limitations as Taylor's formula ; but it bears a very close resemblance to a form previously proposed by Hovgaard,‡ who expressed the total resistance as

$$R = f\, S\, V^{1{\cdot}825} + b\, B^2\, D \frac{V^4}{L^2}$$

where S is the wetted surface, f the frictional coefficient, and b a form coefficient depending on the prismatic coefficient. In arriving at his formula, Hovgaard adduced reasons to show the probability that the total resistance

$$R \propto f\, S\, V^{1{\cdot}825} + p \frac{V^2}{L} + q \frac{V^4}{L^2} + r \frac{V^6}{L^3}$$

the last three terms representing the residuary portion ; and that in order to maintain the law of mechanical similitude or comparison, the coefficients p, q, r, must be functions of L, B, and D, and some

* "Approximate Formulae for Ship Resistance." A. W. Johns, Trans. I.N.A., 1907.

† "Analysis of Ship Resistance." T. Tutin., Trans. I.N.A., 1924.

‡ "Analysis of Resistance of Ships." W. Hovgaard, Trans. I.N.A., 1908.

coefficient of form, such that they will be of the type $k\ L^x\ B^y\ D^z$ provided that always $x+y+z = 3$.

Replacing the two terms $\left(p\ \frac{V^2}{L} + r\ \frac{V^6}{L^3}\right)$ by an equivalent single term $q^1\ \frac{V^4}{L^2}$ his expression becomes

$$R \propto f\ S\ V^{1\cdot825} + (q+q^1)\ \frac{V^4}{L^2}$$

and he finds the Law of Comparison satisfied by making $q+q^1 = b\ B^2\ D$ in which it will be seen the sum of the three indices is equal to three. This formula in its general structure was based on and justified by an examination of model experiments by Froude and Rota, but the value of b has always to be determined by comparison with ships of similar type or by experiment, and where ships have parallel middle body, L must be replaced by the sum of the lengths of the entrance and run.

The general line of thought followed by Hovgaard in arriving at his formula has been indicated because it appears to be the logical road that must be trodden if the results obtained either from ships or models are to be represented in a form which admits of calculation. In view of the extreme complexity of the problem as we now see it, the question naturally arises as to what are the possibilities and limitations of a general formula. It seems to the writer that these can best be seen by an analysis and study of a given set of model experiments treated in the light of the general principles which have been established. The eight principal modes of variation to which reference has been made, open up numerous avenues of approach, and practically compel the systematic investigator to adopt some parent form and then vary it in certain definite prescribed ways in order to assess the relative values of particular elements of variation. This course was taken by the experimenters at the William Froude Tank, to find the effect of beam variation on the resistance of mercantile ship forms. The results appeared in a paper by Kent in the " Trans. I.N.A., 1919." In this series, four parent forms were chosen, which, referred to a length of 400 ft., represented a ship $400' \times 65{\cdot}33' \times 23{\cdot}23'$ draft, but differed in the length of parallel middle body and prismatic coefficient, which were as follows :—

Model.	L.	M.	N.	O.
Per cent parallel body,	0	10	30	50
Prismatic coefficient,	·655	·689	·758	·827
$\frac{L_e}{L_r}$	1·0	1·0	·9	·9

Variations of the beam for each of these parent forms, ranging from about 40′ to 90′ were made under three different restrictions :—

(*a*) Keeping the draft constant ;

(*b*) Keeping $\dfrac{B}{\sqrt{A}}$ constant ;

(*c*) Keeping A constant.

Curves showing the effect of these modes of variation on the C value were given for a series of P values to a base of beam in each case. It should be interesting, therefore, to see how far a general formula can be arranged to meet the results thus obtained, and, in so doing, express the law of variation. Any such relation must be prepared to express that

Total Resistance = Skin Resistance + Residuary Resistance.

Transverse Waves.	Diverging Waves.	Wave Interference.	Eddy Making	Minor Effects.

Starting then from the value of C in terms of total resistance, we must have :—

$$\frac{C \times \triangle^{2/3} \times V^2}{427 \cdot 1} = R.$$

and as any error made in taking skin resistance equal to aV^2 will merely modify the law of the coefficient a, and as there are reasons for supposing that the various forms of residuary resistance involve terms of the order of V^4 and V^6, it is not unreasonable to put

$$R = p\, V^2 + q\, V^4 + r\, V^6$$

which gives $$\frac{C\, \triangle^{2/3}\, V^2}{427 \cdot 1} = p\, V^2 + q\, V^4 + r\, V^6$$

i.e., $$C = p^1 + q^1\, V^2 + r^1\, V^4$$

An examination of Fig. 5 shows the scope of the task, which is to determine the form and nature of the coefficients p^1, q^1, r^1, so that all curves there shown may be reproduced at will from the formula. Two features of the curves immediately strike the eye ; the periodical recurrence of " humps " and " hollows," and the irregular manner in which the curves intermingle and terminate at the lower values of P. The first characteristic requires that the formula should be so constructed as to reproduce this interference effect. The second is an illustration of the instability of the model observations at low speeds, a characteristic which is

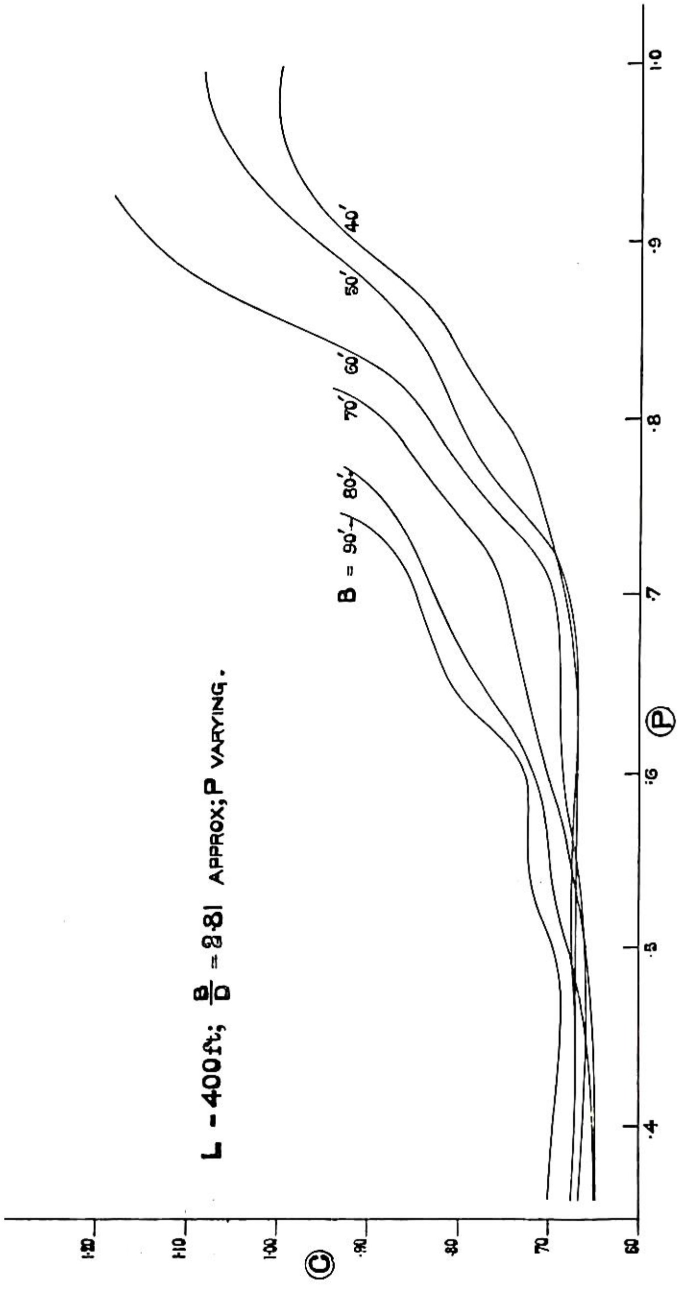

L - 400ft; B/D = 2·81 APPROX; P VARYING.
B = 90'
80'
70'
60'
50'
40'
C
1·20
1·10
1·00
·90
·80
·70
·60
P
·4
·5
·6
·7
·8
·9
1·0

FIG. 5.

to be expected from the results of skin friction experiments already mentioned; in fact, it has been stated that for $\frac{V}{\sqrt{L}} < \cdot 5$ it is very difficult to reproduce identically the observations with models in a tank. In dealing with the curves, some adjustment of each at the lower end may, therefore, be necessary to arrive at the truth.

The proposed formula indicates that as V becomes smaller, Ⓒ tends to a constant value, and this is seen broadly to be the case with the curves in the Fig. 5. It will be assumed that this constant value is reached at Ⓟ $= \cdot 35$, and that it will be possible to build up a curve of Ⓒ value by assuming that the mean ordinate is made up of two portions; C_1 a constant quantity which approximately represents the frictional and quasi-frictional elements of resistance, and C_2 a quantity varying from zero at Ⓟ $= \cdot 35$ and representing the residuary element. The equation for Ⓒ would then assume the form:

$$\text{Ⓒ} = C_1 + C_2$$

$$= C_1 + a\,\text{Ⓟ}_1^{\,2} + b\,\text{Ⓟ}_1^{\,4} \quad \text{Where } \text{Ⓟ}_1 = \text{Ⓟ} - \cdot 35.$$

after expressing V in terms of Ⓟ$_1$, which is virtually in terms of Ⓟ, so as to follow the Law of Comparison.

Now Baker's experimental work has shown that the turning points in the interference effect, that is, the points at which the undulating curve will cross the mean curve, occur for the values:

$$\frac{4}{\text{Ⓟ}^2} = 4,\ 8,\ 12,\ \text{etc.}$$

and that the maximum effects occur for:

$$\frac{4}{\text{Ⓟ}^2} = 5,\ 9,\ 13,\ \text{etc.}$$

and minimum effects for;

$$\frac{4}{\text{Ⓟ}^2} = 3,\ 7,\ 11,\ \text{etc.}$$

If, therefore, the 'residuary' terms in the proposed formula are multiplied by a factor of the form:

$$\left[1 + c \cos 2\pi \left(\frac{1}{\text{Ⓟ}^2} - \frac{1}{4}\right)\right]$$

it will be found that the value of C_2 will become periodic and will result in a curve of the requisite undulatory form as indicated by

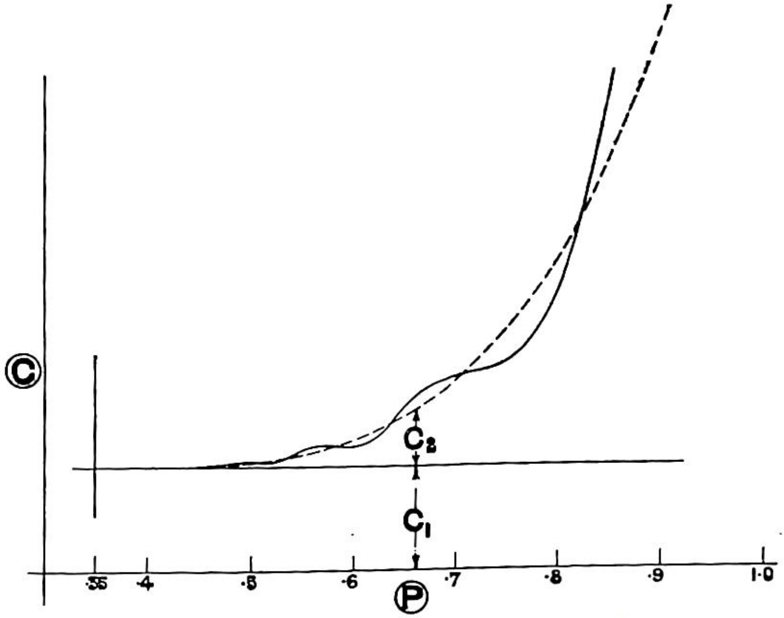


FIG. 6.

the drawn line in the Fig. 6; the amplitude of the "hump" variation being given by the factor c. The formula now becomes :—

$$Ⓒ = C_1 + \left[a\,Ⓟ_1^{\,2} + b\,Ⓟ_1^{\,4} \right] \left[1 + c \cos 2\pi \left(\frac{1}{Ⓟ^2} - \frac{1}{4} \right) \right]$$

and as it stands should be capable of reproducing the main features of the experimentally determined curves.

By assigning the values $C_1 = \cdot 66$; $a = \cdot 10$; $b = 4\cdot 5$; $c = \cdot 20$, the curve L shown in Fig. 7 was produced, and the points indicated by crosses are the values taken from the published results for the parent model L referred to above. This is a very close mean curve except at two localised portions of the range of speed, to which reference will be made later; and even at these points the departure from the experimental values is less than 3%.

In order to pass from the parent model to the other models M, N, O, it was found convenient to write the formula thus :—

$$Ⓒ = \cdot 66\,A + B \left[a\,Ⓟ_1^{\,2} + b\,Ⓟ_1^{\,4} \right] \left[1 + c \cos 2\pi \left(\frac{1}{Ⓟ^2} - \frac{1}{4} \right) \right]$$

For the L model $A = 1$, $B = 1$. The characters of the mean curves for the other models differing, as they did, from that of L, could

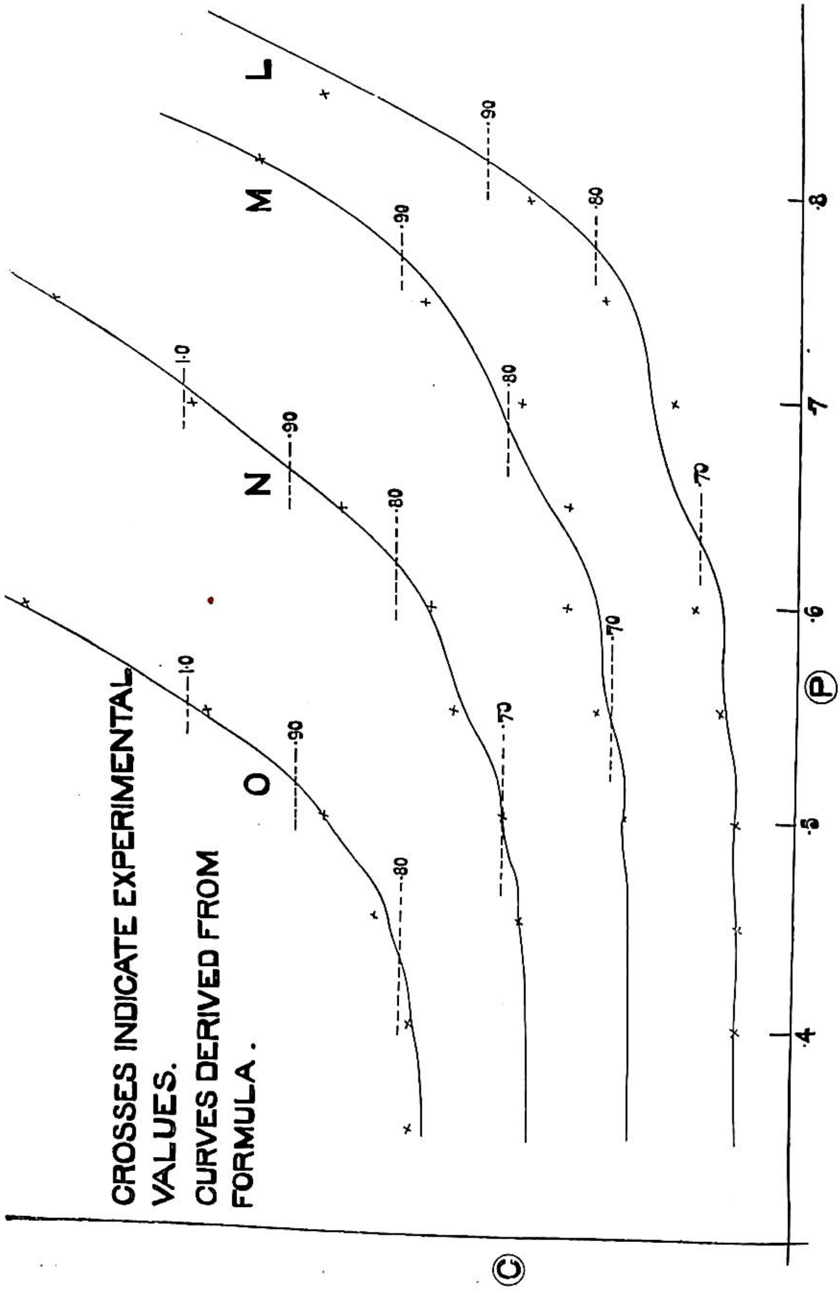

CROSSES INDICATE EXPERIMENTAL
VALUES.
CURVES DERIVED FROM
FORMULA.
L
M
N
O
P
C
·4
·5
·6
·7
·8

FIG. 7.

be reproduced by giving A an appropriate value, keeping $B=1$, and adjusting the factors a and b. These latter were found to be connected with the length of the parallel middle body by relations of the type :—

$$a = \frac{l_1}{m_1 - p} - n_1$$

$$b = \frac{l_2}{m_2 - p} - n_2$$

where l_1 m_1 n_1 etc., are constants, and p is the percentage length of parallel middle body.

The following scheme of values produces the curves, as shown in Fig. 7 :—

Model.		L.	M.	N.	O.
A	=	1·000	1·029	1·013	1·172
B	=	1·000	1·000	1·000	1·000
a	=	·100	·267	·880	3·001
b	=	4·500	6·212	12·800	53·220
c	=	·200	·100	·030	·010

Having produced the basic curve for each parent model, it can be taken as a starting point for some systematic variation. The one chosen for illustration is that made by Kent with the L model in which the beam was varied whilst maintaining a constant value 1·694 for the ratio $\frac{B}{\sqrt{A}}$; this series being selected because the range of (P) value was greater than in the other experiments, and therefore affords a better opportunity of illustrating the scope of the formula. Here it was found possible to closely reproduce the observed values by varying A, B, and c, whilst a and b kept the same values as for the parent model; the series of curves generated from the formula are shown in Fig. 8, the maximum deviation from the observed values being of the order of 2%, except at one special range, to which further reference is made. The variation of the quantities A, B, and c are illustrated in Fig. 9, where their values are plotted on a base of 10^{-4} P $B^2 \sqrt{A}$; this being a function which, as previously noted, falls in with the Law of Comparison. This Figure may be looked upon as a characteristic diagram for this mode of variation from the parent model, and, when used in conjunction with the formula, will give the (C) value for any intermediate model of this type within the range of the variation.

Similar characteristic diagrams can be made which, with the same formula, will give the variations of the other three parent

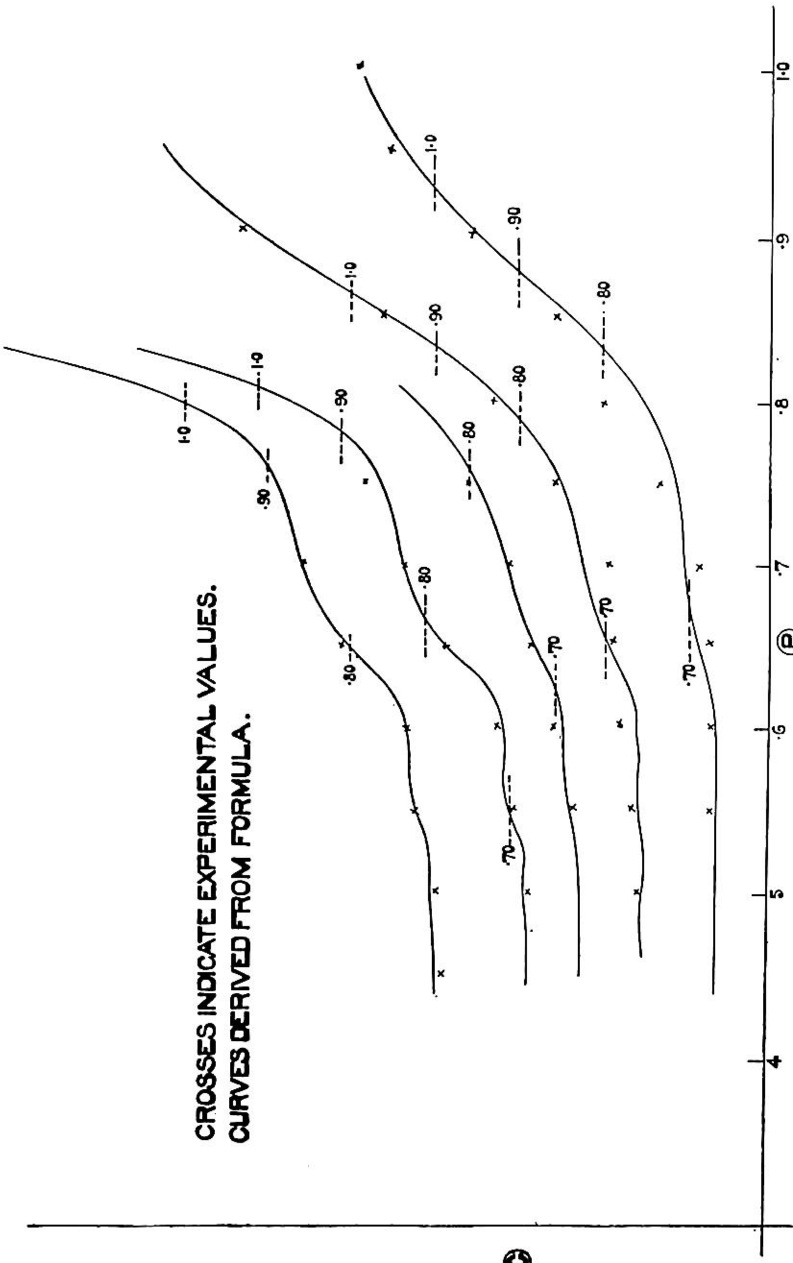

CROSSES INDICATE EXPERIMENTAL VALUES.
CURVES DERIVED FROM FORMULA.
Ⓟ
Ⓒ
·4
·5
·6
·7
·8
·9
1·0

FIG. 8.

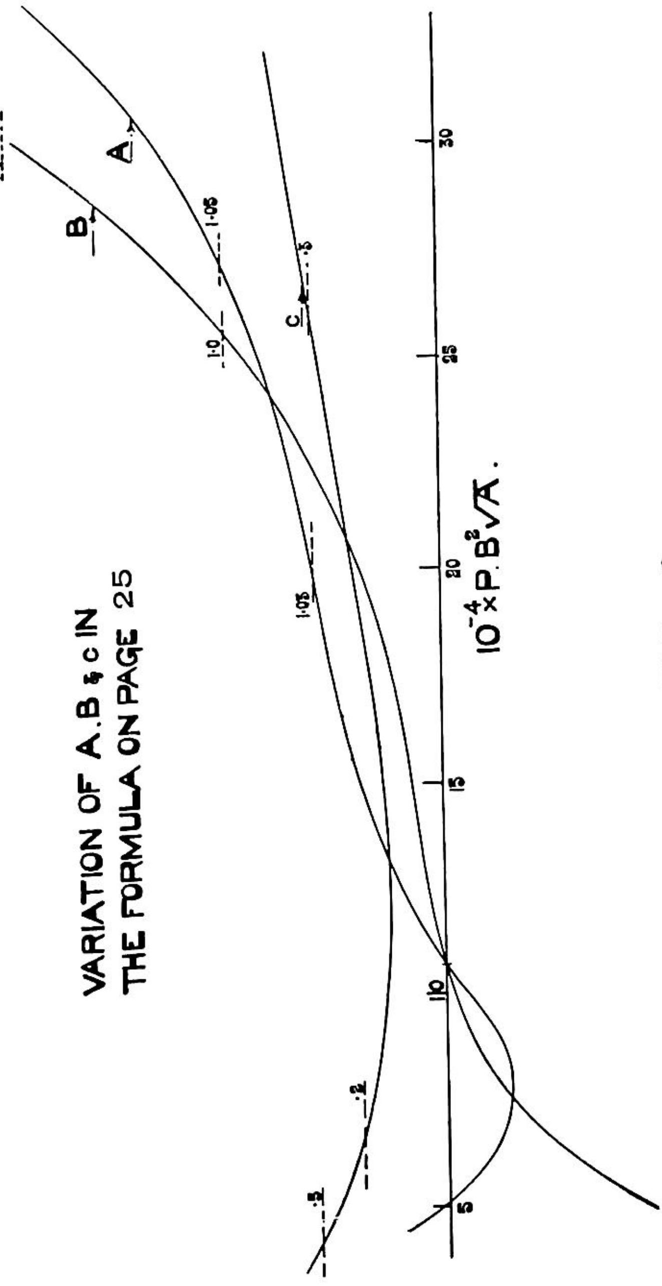

VARIATION OF A.B & c IN
THE FORMULA ON PAGE 25
A
B
C
1·5
1·05
1·0
·5
1·05
·2
·5
5
10
15
20
25
30
10⁻⁴ × P.B² √A.

FIG. 9.

models with equal closeness. In the Figure for the L model variations, it will be noticed that for the lowest pair of curves the observed spots depart from the formula curve over the range (P) = ·65 to ·85 in such a fashion as to suggest that the main interference effect in this region had been disturbed by a subsidiary interference effect. Such a subsidiary interference phenomenon has been demonstrated by Baker to have a real existence (Trans. I.N.A. 1913), when the length of entrance bears a certain relationship to the speed of the ship, and the position on the curves at which this disturbance occurs would seem to show that something analogous is the explanation in this instance. An interesting point is that this irregularity appears to die away as the beam is increased, so that the upper curves in Fig. 8 show only the effects of the main wave interference. Whilst the formula under consideration is not one by means of which the (C) value for any given ship can be estimated without reference to a parent form and a particular mode of variation, it does embody the main factors which experiment has shown to operate in the modification of ship resistance ; and it is suggested that it provides a means of fairing up and analysing experimental data, so that by means of characteristic diagrams, such as that illustrated above, a close approximation can be made to any case within the range of interpolation. The fact that the results of such a wide range of experiments, apart from the subsidiary interference effects mentioned, can be brought within the framework of a single algebraic expression shows the possibilities of making the resistance assessment amenable to calculation, whilst the divergences shown in the diagrams emphasise the lurking dangers attendant on any blind following of rigid formulae. It should be remembered, of course, that very rarely is the designer at liberty to shape the hull with sole regard to the speed qualities ; necessities of providing ample deck areas, or the limitations of trim conditions and the like, as a rule restrict his freedom in this respect. The value, therefore, of published researches, such as those which have been used above, cannot be denied as affording a means by which an assessment of particular variations from some known form can be made.

Having by this or other means placed a value on the resistance likely to be experienced by the ship's hull, the equally important question arises as to the provision of the thrust to overcome this resistance by means of the propeller, and the determination of the essentials for the successful performance of the latter.

(b) THE PROPELLER AND ITS PERFORMANCE.

Until recently there has been no adequate propeller theory on which direct calculations could be based. The theories of the Froudes, of Rankine, Greenhill, and others all assumed different

modes of action by which some explanation of the thrust in terms of the motion of the water could be made. Developments in Aeronautical Science have given us, however, a clearer conception of just how the blade of a propeller acts on the water which it meets, and, although the aerofoil circulation theory has not yet come within the domain of use for rapid estimating, it presents possibilities which the older theories did not afford. The only practical basis of action at present is provided by the results obtained from model experiments linked up with the analysis of the performances of full-size screws, where these are available. The conditions under which model propeller experiments can be made to yield useful information have already been explained; and there is now a wealth of published data to which the designer may turn for guidance. This is the result of the extensive work carried out in England, America, and Germany, by such investigators as Froude, Taylor, Durand, Gebers, Schaffran, and others, who have verified the validity of the Law of Comparison by experimenting with screws of different sizes. Taylor, for example, compared the results of similar propellers 8-in. and 24-in. diameter under similar conditions; Gebers compared 3-in. and 12-in. screws; and at the National Physical Laboratory propellers 2-ft. and 15-ft. diameter were tested in air on a revolving arm. In every case the Law was found to hold within a small percentage error. So far, therefore, as concerns propellers working in open water away from the influence of the hull of the vessel, the results obtained from this experimental work may be applied without fear of any material discrepancy; in fact, the systems of calculation given by R. E. Froude, D. W. Taylor, and others, enable the principal characteristics of a propeller of the type to which their systems refer to be determined very quickly. Just as in the case of the hull form, however, there are a variety of ways in which the actual ship propeller may differ or be compelled to differ from the assumed propeller of the experiment working under ideal conditions.

The Law of Comparison puts us in a position to say that :—

$$\frac{T}{D^2 V^2} = c$$

where T is the thrust; D is the diameter; V the speed of advance of the propeller through the water in which it works; and c a constant as between model and full size screw. Experiment demonstrates also that c is a function of the "slip" and the design of the particular propeller. The slip depends on the pitch ratio and revolutions; thus, the three main features are undoubtedly the pitch, diameter, and revolutions. But the shape and the rake of the blades, the disc area ratio, the thickness and shape of the blade section, the position of the maximum thickness in the section, the hollowness of the driving face, and the roughness of the surface, all have an

effect on the efficiency of the performance, which may be thus affected in many hundreds of ways by the modification of these different factors.

From the point of view of the powering of the ship, however, only one side of the propeller problem has been attacked by the model experiments in open water, and the effect of the hull on the performance of the propeller is, in the issue, the controlling element of the case ; for the design of the screw on the results of open water experiments assumes the inflow of the water to the propeller disc to be a steady uniform stream parallel to the direction of advance, and this is by no means true when the screw is in position behind the vessel. To make some allowance for this difference, the usual assumption has been that the uniform stream has had its velocity of inflow to the propeller diminished by a definite percentage measured by the wake fraction w where

$$\frac{V}{V_1} = 1 + w$$

V_1 being the new diminished velocity. The resistance to be overcome has also been assumed to be reduced owing to the action of the propeller, the effect being measured by the quantity t, known as the "thrust deduction factor," where

$$\frac{R}{T} = 1 - t$$

T being the propeller thrust and R the resistance.

But experiments, and experience with full-sized ships have shown that these modifications are not sufficient to explain the results obtained, and that, to trace the relationship between the effective horse power (E.H.P.) and the shaft horse power (S.H.P.), there are at least three efficiencies to be taken into consideration.

The sequence of the losses from the S.H.P. measured at the propeller to the E.H.P. delivered to the ship is symbolically and conveniently represented thus :—

$$\frac{\text{E.H.P.}}{\text{S.H.P.}} = \frac{RV}{S} = \overset{(1)}{\frac{R}{T}} \times \overset{(2)}{\frac{V}{V_1}} \times \overset{(3)}{\frac{TV_1}{S_1}} \times \overset{(4)}{\frac{S_1}{S}}$$

$$= \text{the propulsive efficiency.}$$

in which

(1) $\frac{R}{T} = 1-t$ where t = thrust deduction factor.

(2) $\frac{V}{V_1} = 1+w$ w = wake fraction.

(3) $\frac{TV}{S} = e_P$ = efficiency of propeller in open water.

(4) $\frac{S_1}{S} = e_R$ = relative rotative efficiency.

It has been found necessary to introduce the fourth link termed " Relative Rotative Efficiency," because experience has shown that when the propeller is working behind the ship, it requires a somewhat different shaft horse power to develop the same thrust to what it does when working in open water.

It will be seen that the product of the terms (2) and (3) is :—

$$(1-t)\ (1+w) = \frac{RV}{TV_1} = \frac{EHP}{THP} = \text{Hull efficiency} = e_H$$

hence the propulsive efficiency becomes :—

$$= e_R \times e_P \times e_H$$

and if the mechanical efficiency of the engine is represented by the symbol e_M then the total overall efficiency e measuring the proportion of the power generated in the engine which is delivered in useful work is expressed by

$$e = e_M \times e_R \times e_P \times e_H$$

The factors in this equation which involve most uncertainty are e_H and e_R. The determination of e_H turns on the assessment of the wake value w, and to this end a great deal of experimental work has been directed. Perhaps the most notable contribution to our knowledge of it is the experimental work carried out by Luke,* who has shown that both t and w, and hence e_H, are affected by varying the speed, the pitch ratio, the diameter, the position relative to the hull, and the slope of the bossings in the case of twin screws. The variation in e_H is comparatively small in both single and twin-screw ships for variation in $\frac{V}{\sqrt{L}}$ and pitch ratio, but it becomes very marked for variation in diameter, and is more sensitive to this in single screw ships ; the alteration of the position athwartships with twin screws is important, but the fore and aft position may be altered without the effect being very serious. One interesting fact which emerged from these experiments showed very clearly that the " wake " was not an inherent property of the hull as such ; it was found that in the case of twin screws with bossings at a given slope, the wake value was widely different according as the screws turned inwards or outwards ; the difference gradually

* " Experiments in Wake and Thrust Deduction." W. J. Luke, Trans. I.N.A., 1910, 1914, 1917.

increasing from zero value when the slope of the bossing was about 30° to 20% when the bossing was horizontal, and to about 24% when it was vertical.

The wake varies also with the relative fineness of the ship, Luke finding a practically linear relationship with the block coefficient, and recent experiments* would seem to show that it is not only the fulness or fineness of the after body which is concerned, but that the fore body also contributes its influence. As another instance of the interlocking of the system, it should be noticed that when the resistance R is increased from any cause whatever, there will be a corresponding increase in the value of t, and an alteration in e_H. Baker found,† for example, in one case that a 30% increase in R caused a 4% increase in t, and at the same time w became less by 3%, thus making a total loss of about 7% in the value of e_H. The wake value is also sensitive to the amount of immersion of the screw. There are, of course, numerous formulae in existence, based more or less on model experiments, which purport to give approximate values for the wake, and the use of Luke's results will, no doubt, give a close figure within the range of his experiments, so far as the "model wake" is concerned, but it is recognised by all who have worked in this field that the "model wake" differs from the wake of the full-sized ship, and Luke, in particular, has insisted on the value of carefully-conducted ship trials for which the Model experiment results simply form "a sieve through which the results of the actual ship trials may be sifted." This being so, it is not difficult to understand that the allowance for wake made in the usual manner does not completely fill the necessities of the case for the determination of the various efficiencies connected with the propeller, and that the introduction of the term e_R has been deemed necessary. Although the value e_R was supposed at one time to differ only by 1% or 2% from unity, recent work in connection with the efficiency of propulsion of full-sized ships shows that when the thrust as well as the torque is measured on the actual vessel, a value as low as ·85 may be necessary in order to bring the observations into consonance.‡ The only effective means of attaining a measure of certainty in the matter is by more extensive measurements of thrust and torque of full-sized ships, and until such information is available, the general designer must be content to forge this link of the chain as best he may.

A very useful and interesting turn has been given to the situation recently by D. W. Taylor,§ who, by a careful comparison of the

* "Self-Propelled Models." E. M. Bragg. Amer. Soc. N.A. and Mar. E., 1924.

† "Model Screw Propeller Experiments." Baker & Wigley., Trans. I.N.A., 1923.

‡ "Efficiency of Propulsion of Full-Sized Ships." C. F. Holt. Trans. I.N.A.," 1920.

§ "Comparison of Model Propeller Experiments, etc." D. W. Taylor. Amer. Soc. N.A. and Mar. E., 1924.

experimental work carried out by Froude, Schaffran, Durand, and himself, found that the results given by these different experimenters were in sufficient agreement for practical purposes, to justify him in producing average or composite diagrams for the combined work. Schaffran* has also made an independent comparison of his own work with that of Taylor, and finds it in close agreement.

Proceeding a step further than the composite diagram, Taylor has proposed a method† which, used in conjunction with the established results of model propeller experiments, gives a means of analysing the usual trial data, and deriving from it a coefficient of propeller performance which can in turn be used in propeller design.

The basis of his investigation is the observed fact that the "capacity for power-absorption" of propellers used in practice may be expressed over a wide range of slip by a formula involving known dimensions and revolutions and a single coefficient; the wide range including the slips usually found in practice. Denoting this coefficient by C_p, his fundamental formula becomes :—

$$P = \frac{C_p}{1-s} \times \left(\frac{p\,R}{1000}\right)^3 \times \frac{d^3}{p}$$

where P, R, d, p, s, refer to shaft horse power, revolutions, diameter, face pitch, and slip respectively.

This was connected with the ship performance by means of a "Wake Propeller Coefficient," W_p.

Noting that for a standard model elliptical-bladed propeller, the value of C_p remained practically constant for the usual working range of real slip ratio, this standard model value of C_p was denoted by S_m, and values of it for the range of pitch ratios likely to be used in practice determined. In order to allow for the various sources of error in fixing the wake value and other factors, the coefficient W_p was connected with the power absorption coefficient by means of the relations :—

$$C_p = k\,S_m$$

$$W_p = \frac{k}{1-w}$$

the equation for the propeller determination then assuming the form :

$$W_p = \frac{1{,}000\ P\,V}{S_m\,d^3 p^3 \left(\frac{R}{100}\right)^4}$$

Taylor gave curves of the value of W_p based on an analysis of the results of 100 twin-screw vessels and 50 single-screw, which may be

* "Vergleich der Propellerversuchsergebnisse." K. Schaffran. "Schiffbau," 22/4/25.

† "Wake Propeller Coefficients." D. W. Taylor. Trans. I.N.A., 1925.

used for designing purposes in the absence of particular values of this coefficient obtained from actual trials which may have come within the designer's own experience. The great convenience of this method of attack, particularly in the early stages of design, is not easily over-rated, especially from the point of view of the assessment of the power, and the present writer has found that this particular line of approach has the additional advantage that it can all be embodied in one diagram which permits the elements of the propeller and their relation to the speed, power, revolutions, and efficiency, to be considered without calculation, and with only a few minutes' expenditure of time. Such a diagram is illustrated in Fig. 10. It has been prepared by collating the information given by Taylor, and applies in the first instance to his ordinary standard propellers having elliptical blades, with a mean width ratio = ·25 ; ratio of developed area to disc area = ·382 ; and blade thickness fraction = ·05.

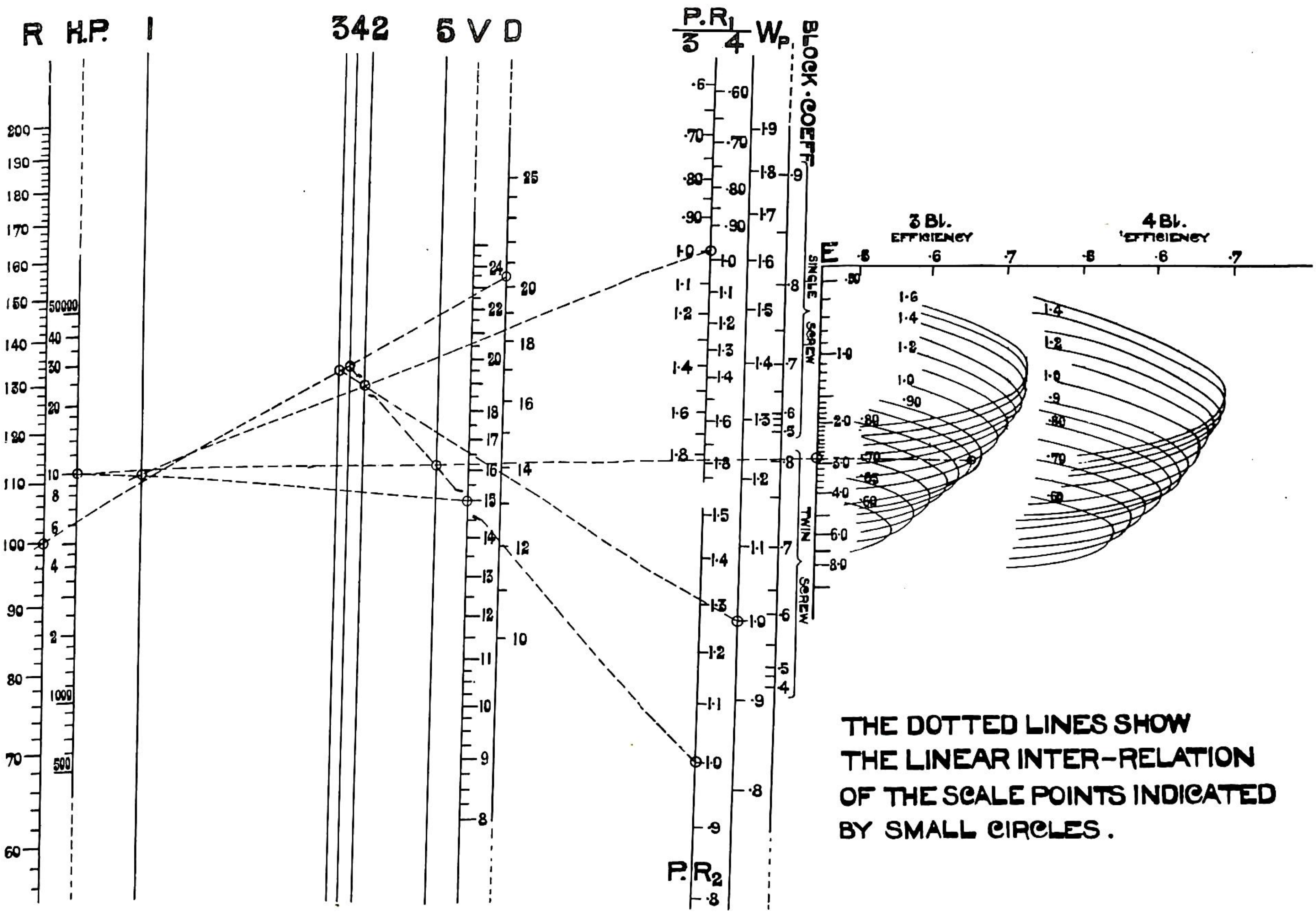
In connection with the application of this formula to propeller design, Taylor has remarked very cogently that " Once wake fraction, thrust deduction, and revolutions, are fixed, a designer, design he never so well, is surrounded by barriers to efficiency, which, so far as our present knowledge goes, are insurmountable by variations of propeller design. The real gain in propulsive efficiency which involves not only the propeller efficiency, but also the whole efficiency factor, has to be sought through control of wake fraction and thrust deduction, and the adoption of revolutions which enable the power to be efficiently developed at the speed of the ship," and this dictum would seem to contain the essence of the truth about propeller design.

To complete the mechanical cycle, it remains to point out one or two features in connection with the prime mover which affect the application of the power generated.

(c) THE PRIME MOVER.

The consideration of the prime mover brings the designer at once face to face with the fact that each type, on whatever principle it is worked, has a certain range of revolutions over which it will work with a maximum of economy, and that, if a departure is made from this range, a depreciation in efficiency will inevitably take place. Hence the importance of always considering the propeller revolutions when fixing the power, and making allowance for any deviation from the economical range.

If reducing gear of any type is introduced in order to produce good propeller revolutions, the efficiency of the gear becomes a factor in the case.



THE DOTTED LINES SHOW THE LINEAR INTER-RELATION OF THE SCALE POINTS INDICATED BY SMALL CIRCLES.

FIG. 10.

See Page 45.

In reciprocating engines there is also the general mechanical efficiency to be considered, and this, although known within a limited range of error, is never exactly known under running conditions. With turbine installations comes the question of the measurement of the shaft horse power, and it has been found that determinations of it by torsionmeter are not reliable as a matter of observation unless the utmost care is exercised in taking the readings; especially under service conditions is it difficult to obtain a record of the performance. Even if the readings are observed to within, say, 1%, the result is still open to question on account of the uncertainty of the coefficient of rigidity of the shaft, which may not be accurately assessed within a range of 4%.* The indicator diagram itself is not above suspicion, so that, whether indicated or shaft horse power is being dealt with, there is an element of doubt even in the measurement of the power which reflects itself in the over-all efficiency, and an error involved which cannot be fixed with precision, and again must find its allowance in the general coefficient of performance.

SERVICE CONDITIONS.

Not infrequently it is necessary, after estimating the power for a practically still-water performance, to determine some margin of allowance which will take account of sea conditions. The indefiniteness of the latter term, and the general manner in which, until recently, it has been treated, are well reflected in the fact that one authority† gives an allowance of something not greater than 15% in power for ships making Eastern voyages, and anything from 30% to 40% for North Atlantic work; whilst another authority‡ quotes a general figure of 25% increase in power to cover for average speed under average weather conditions. The difference of interpretation by different designers of the general conditions which an owner may have in mind, may therefore be very great, and the desire to have more precise knowledge on the subject has led the Authorities of the National Physical Laboratory in recent years to make some systematic investigation of the effect of wind and waves on the propulsion of ships.

The observations at sea were preceded by a series of experiments in the William Froude Tank in which models were run in waves artificially created, whose lengths and heights were under the control of the operator.§ The experiments, amongst other results, showed that the loss of speed for a given horse power varied approxi-

* *Vid.* Appendix to First Report of Marine Oil-engine Trials Committee, 1924.
† Sir A. Denny. Discussion on paper by J. L. Kent. Trans. I.N.A., 1924.
‡ Sir J. Biles. Discussion on paper by J. L. Kent. Trans. I.N.A., 1924.
§ "Experiments on Merchant Ship Models in Waves." J. L. Kent. Trans. I.N.A., 1924.

mately as the height of the waves, and that as the waves increased in length, the resistance became more pronounced; the percentage increase in power for the same wave height remaining practically constant, except when the natural pitching period of the vessel was equal to the "encounter" period of the waves. It was obvious, of course, that there would be differences between these results obtained from the naked model and those that would be obtained from an actual ship at sea, if only from the fact that the ship would be propelled by a screw, and that uniform waves are seldom met with on the ocean. To ascertain at first hand how far the model results would be confirmed or otherwise under sea conditions, Mr. Kent of the William Froude Tank made a number of voyages and investigated, under very diverse weather conditions, the behaviour of four vessels, the "Montcalm," "London Mariner," "San Gerardo," and "San Tirso."* The observations were remarkably complete, considering the attendant conditions and difficulties. He found four principal causes of reduction in speed:

(*a*) Waves. (*c*) Loss of propeller thrust.
(*b*) Wind. (*d*) Increase of resistance due to steering.

He confirmed that, as found in the model experiments, the wave resistance depended principally on the height and the "encounter" period, and if the ship's natural period of pitching was equal to the "encounter" period, then heavy pitching ensued, causing a still further loss. The wind resistance increased approximately as the square of its apparent forward velocity. As the shaft horse power and displacement were continually varying, he plotted his observations in the form of curves of C_s value on a base of speed, where :—

$$C_s = \frac{\text{S.H.P. } 427{\cdot}1}{\triangle^{2/3} \times V^3} \text{ corrected to a standard temp. of } 55^\circ\text{F.}$$

A series of curves were drawn for $\dfrac{\text{S.H.P.}}{\triangle}$ = constant, each curve corresponding to a C_s value at one speed under fine weather conditions for a particular $\triangle$ value, and at a given S.H.P. As the weather became worse, the appropriate value of C_s would lie on the particular $\dfrac{\text{S.H.P.}}{\triangle}$ curve. From an analysis of the results, it appeared that in the case of the "Montcalm," for a particular value of $\dfrac{\text{S.H.P.}}{\triangle}$ the total loss of power varied from 12% to 26%, and in another case, for the "San Gerardo," the limits were 2% to 31%.

* "Effect of Wind and Waves on the Propulsion of Ships." J. L. Kent. Trans. I.N.A., 1924.

A careful inspection of these curves, however, will reveal the fact that they are all hyperbolic in form ; so that the whole variation associated with any particular value of $\frac{\text{S.H.P.}}{\Delta}$ is represented by $C_s\ V^3 = k$ a constant. If now the values of k are plotted as abscissae with the corresponding values of $\frac{\text{S.H.P.}}{\Delta}$ as ordinates, it will be found that the series of spots, each representing one curve, will lie on a straight line passing through the origin.

Typical straight lines for three of the vessels for which observations were taken by Mr. Kent are shown in Fig. 11. It is obvious, of course, that a similar straight line diagram is obtained if instead of $C_s\ V^3$ and $\frac{\text{S.H.P.}}{\Delta}$ being used as ordinates, the value of $\sqrt[3]{\frac{\text{S.H.P.}}{\Delta}}$ is plotted on a basis of V, assuming the practical constancy of C_s over a given limited range.

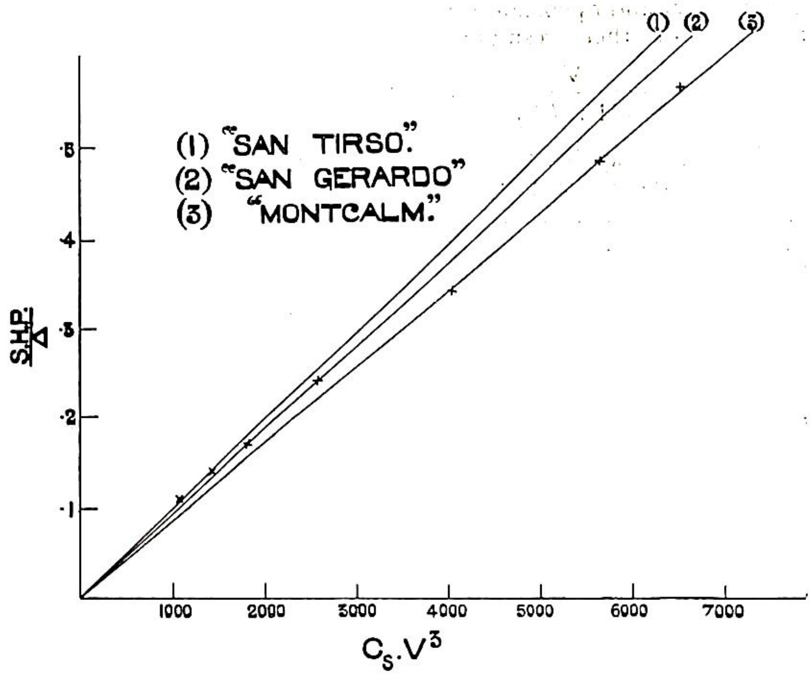


FIG. 11.

This gives a very convenient and useful diagram for the comparison of speed performances of a given vessel; because if one spot on the straight line is known either from trial or other smooth water conditions, then the spots obtained from sea observation will enable a close estimate to be made of the sea speed; or, on the other hand, if several sea observations are taken and a straight line drawn through the origin and the centroid of the spots, a close estimate of the real sea speed and performance is obtained.

This result is the same as that recently pointed out by Schaffran,* who arrived at it by a different line of argument, and recommended its use because of the extreme variations and uncertainties attached to the determination of the sea speed, and the large errors involved with the usual formulae containing V^3 as a factor. For he argues that

$$\text{if} \quad \frac{R}{\triangle} = c_1 \text{ and } \frac{V}{\sqrt{L}} = c_2$$

$$\text{then} \quad \frac{\text{S.H.P.}}{\triangle V} = c_1 \text{ and } \frac{\triangle V^3}{L \times (\text{S.H.P.})} = \frac{c_2^{\,2}}{c_1} = C_s^1 \text{ say;}$$

hence, for c_1 and c_2 constant we see that for the same ship in which L = constant there results:

$$\frac{V}{(\text{S.H.P.}/\triangle)^{1/3}} = \sqrt[3]{C_s^1} = \text{Constant.}$$

In concluding this short sketch, the writer hopes that the main aspects of the problem have been made sufficiently clear to demonstrate the impossibility of exactitude, but at the same time to show the possibilities which are now open to the designer to enable him to form his judgment.

It will also be evident that for the successful powering of any vessel, there is need for close co-operation by all concerned in the design and construction of the engines, propellers, and hull, in order to obtain the most satisfactory result, and that there is need for the field of experimental research to be still further widened, so that data may be available on which the effects of all elements of the problem can be assessed, even if the modifying causes cannot be fully controlled.

* "Die Anwendung der Admiralitäts—oder Leistungskonstanten u.s.w." K. Schaffran. "Schiffbau," 13/5/25.

APPENDIX I.

The Froude "Constant" Notation.

1. $\text{M} = \dfrac{L}{\triangle^{1/3}} \times \cdot 3057$ See Fig. 3.

2. $\text{S} = \dfrac{S}{\triangle^{2/3}} \times \cdot 0935 = 3 \cdot 4 \times \dfrac{\text{M}}{2}$ approx.

3. $\text{K} = \dfrac{V}{\triangle^{1/6}} \times \cdot 5834$

$\text{K} = \text{L} \times \text{M}^{\frac{1}{2}}$

4. $\text{L} = \dfrac{V}{\sqrt{L}} \times 1 \cdot 055$

5. $\text{P} = \dfrac{V}{\sqrt{P\,L}} \times \cdot 746$

See Fig. 2.

Frictional. Residuary.

6. $\text{C} = \dfrac{\text{E.H.P.}}{\triangle^{2/3} \times V^3} \times 427 \cdot 1 = \text{F}_s + \text{C}_r$

See Fig. 4.

The corresponding values of the speed-length constants K, P, L, and the value of $V/\sqrt{L}$, can be easily read from Fig. 2, when V, L, and P are known.

In conjunction with this, Fig. 4 provides the means of determining by collinearity of points :—

(*a*) The value of I.H.P. or S.H.P. for given values of $\triangle$, V, and C, the ordinary Admiralty "Constant."

(*b*) The value of E.H.P. for given values of $\triangle$, V, and C.

(*c*) The amount of E.H.P. due respectively to given values of F_s and C.

(*d*) The value of the Resistance in lbs. per ton of Displacement $\dfrac{R}{\triangle}$ for any given values of

C & K; C_r & K; F_s & K.

The value of (F)$_s$ can be readily obtained from Fig. 3, where the quantity a is determined from Fig. 3a.

APPENDIX II.

Notes on the Diagrams.

Fig. 2.

The Scales are arranged to give the speed values on the " Constant " System where :—

V = Speed in knots.
L = Length of ship in feet.
$\triangle$ = Displacement in tons.
P = Prismatic coefficient.

(K) is simply obtained by joining the values of $\triangle$ and V, and reading the value at the intersection on the (K) line.

(L) and $\dfrac{V}{\sqrt{L}}$ have corresponding values on the same scale-line, and the straight line joining the given values of V and L intersects the (L) line at the corresponding value of (L).

(P) is derived from (L) by joining the (L) point to the given point on the P scale and producing the line to cut the (P) scale.

To reproduce the Figure on a size which has been found convenient for ordinary working purposes, the individual scales may be taken as follows :—

log V ($1'' = \cdot 05$) ; log (K) ($1'' = \cdot 0625$) ; log (P) ($1'' = \cdot 04$) ; log. (L) ($1'' = \cdot 10$) ; log $\triangle$ ($1'' = \cdot 075$) ; log L ($1'' = \cdot 10$) ; log P ($1'' = \cdot 04$).

The corresponding distances between the scales should be :—

V to (K) $= 1''$; (K) to (P) $= 1{\cdot}5''$; (P) to (L) $= 1{\cdot}5''$; (L) to $\triangle = 1''$; $\triangle$ to L $= 3''$; L to P $= 2''$.

Figs. 3 and 3a.

The Scales are arranged to give the values (M) and $(F)_s$ on the "Constant" System where :—

V = Speed in knots.
L = Length of ship in feet.
$\triangle$ = Displacement in tons.
a, F } Are coefficients connected with (M) and $(F)_s$ by the relation :

$$(F)_s = a\,F\,(M)^{\frac{1}{2}}$$

The value of a is determined by Fig. 3a, from which it will be seen to depend on the midship area coefficient and the ratio of the beam to the draft.

The case illustrated in Fig. 3 is for a ship in which :—

$$\triangle = 18{,}000\ ;\ L = 500\ ;\ a = 2{\cdot}7\ ;\ \frac{V}{\sqrt{L}} = {\cdot}70.$$

The line $\triangle - L$ gives (M) $= 5{\cdot}83$; the line $L - V/\sqrt{L}$ gives $F = {\cdot}076$; if now the line joining the value $a = 2{\cdot}7$ to the point in which (M) $-$ F intersects the support line be produced, it will give the appropriate value of $(F)_s = {\cdot}496$.

To reproduce the figure on a size which has been found convenient for ordinary working purposes, the individual scales may be taken as follows :—

log L ($1'' = {\cdot}048$) ; log (M) ($1'' = {\cdot}072$) ; log $(F)_s$ ($1'' = {\cdot}060$) ; log F ($1'' = {\cdot}036$) ; log $\triangle$ ($1'' = {\cdot}072$) ; log a ($1'' = {\cdot}012$).

The $\dfrac{V}{\sqrt{L}}$ scale to be arranged of same relative size as shown in the figure.

The corresponding distances between the scales should be :—

L to (M) $= 2''$; (M) to $(F)_s = {\cdot}9''$; $(F)_s$ to 'support' line $= 1''$; 'support' to $\dfrac{V}{\sqrt{L}} = 1{\cdot}1''$; $\dfrac{V}{\sqrt{L}}$ to $F = {\cdot}8''$; F to $\triangle = {\cdot}2''$; $\triangle$ to $a = 2{\cdot}9''$

Fig. 4.

The Scales are arranged to give :—

(*a*) The I.H.P. or S.H.P. corresponding to an assumed value of the Admiralty Constant C ; for this the scale marked H.P. is to be used.

(*b*) The E.H.P. corresponding to any pre-determined value of Ⓒ, $Ⓒ_r$, or $Ⓕ_s$; for this the scale marked E.H.P. is to be used.

(*c*) The Resistance in lbs. per ton of Displacement corresponding to any given value of Ⓚ and a given value of Ⓒ, $Ⓒ_r$ or $Ⓕ_s$.

Where R = Resistance in lbs.
$\triangle$ = Displacement in tons.
V = Speed of ship in knots.

The case illustrated in the figure shows that for :—

$\triangle = 15{,}000$; $V = 18$; $C = 272$; the I.H.P. or S.H.P., as the case may be, is 13,000. Or if a Ⓒ value of ·845 has been determined, then the E.H.P. = 7000. The Ⓚ value as found by Fig. 2 being 2·12, the value 8·3 for the Resistance in lbs. per ton of Displacement can be read on the $\frac{R}{\triangle}$ Scale.

To reproduce the figure on a size which has been found convenient for ordinary working purposes, the individual scales may be taken as follows :—

log $\triangle$ (1″ = ·15) ; log Ⓒ &c. (1″ = ·06) ; log C (1″ = ·06) ; log $\frac{R}{\triangle}$ (1″ = ·18) ; log Ⓚ (1″ = ·06) ; log E.H.P. or H.P. (1″ = ·24) ; 3 log V (1″ = ·20).

The corresponding distances between the scales should be :—

$\triangle$ to Ⓒ &c. = 2″ ; Ⓒ &c. to $\frac{R}{\triangle}$ = 2″ ; $\frac{R}{\triangle}$ to Ⓚ = 1″ ; Ⓚ to 'support' = 1″ ; 'support' to E.H.P. or H.P. = 1″ ; E.H.P. to V = 2″.

Fig. 10.

The Scales are arranged to give :—

Revolutions per minute, ...	R
Horse-power delivered to Propeller, ...	H.P.
Speed of Ship in knots, ...	V
Diameter of Propeller in feet, ...	D
Pitch Ratio for 3-bladed Propeller, ...	$\frac{P\,R_1}{3}$
Pitch Ratio for 4-bladed Propeller, ...	$\frac{P\,R_1}{4}$
Pitch Ratio auxiliary scale, ...	$P\,R_2$
Wake Propeller coefficient, ...	W_p
Block coefficient for Single-Screw Ship, ...	
Block coefficient for Twin-Screw Ship, ...	
Efficiencies e of Propeller for various pitch ratios plotted on a base E for three-bladed and four-bladed screws.	

The support lines of the diagram are indicated by the figures 1, 2, 3, 4, 5, at the top of the respective lines.

The dotted lines show the inter-relation of the various quantities for the case of a three-bladed Propeller, for which :—

$V = 15$; $R = 100$; H.P. $= 10{,}000$; P.R. $= 1{\cdot}0$;
$W_p = 1{\cdot}0$ for Twin Screw with block coefficient ·58.

The intersection of HP—V with support 1 is joined to $\frac{P\,R_1}{3}$ by a line intersecting support 2 in a point which is then joined to W_p. The line W_p—2 is produced to intersect support 3 ; then R—3 will intersect scale D at the point which gives the appropriate diameter. The point in which the line R—3 intersects 4 is then joined to the pitch ratio value on the scale P.R.$_2$ and the point where this line 4—P.R.$_2$ cuts the support 5 is joined to the H.P. value ; the line H.P.—5 is then produced to cut the E scale, and the appropriate efficiency can be read off from the curve corresponding to the P.R. value. In the case considered, the group of correlated values becomes :

H.P. $= 10{,}000$; $R = 100$; $V = 15$; $W_p = 1{\cdot}0$; P.R. $= 1{\cdot}0$;
$D = 20{\cdot}4$; $e = {\cdot}67$.

The efficiency curves on the diagram apply strictly only to Taylor's standard elliptical bladed propeller, with a mean width ratio $=\cdot 25$, a ratio of developed area to disc area $=\cdot 382$, and a blade thickness fraction $=\cdot 05$. They are, however, sufficiently general in their application to form a guide in assessing the power in relation to the Propeller performance.

To reproduce the Figure on a size which has been found convenient for working purposes, the individual scales may be taken as follows :—

log R ($1'' = \cdot 05$) ; log H.P. ($1'' = \cdot 30$) ; log V ($1'' = \cdot 06$) ; log D ($1'' = \cdot 06$) ; log PR_1, log PR_2 ($1'' = \cdot 05$) ; log W_p ($1'' = \cdot 04$) ;

Block Coefficient Scales to correspond with W_p scale as on diagram ; log E ($1'' = \cdot 30$) ; efficiency ordinates ($1'' = \cdot 10$).

The corresponding distances between the scales should be :—

R to H.P. $= \cdot 5''$; H.P. to 'support' $1 = 1''$; 'support' 1 to 'support' $3 = 3''$; 'support' 3 to 'support' $4 = \cdot 15''$; 'support' 4 to 'support' $2 = \cdot 25''$; 'support' 2 to 'support' $5 = 1\cdot 10''$; 'support' 5 to V $= \cdot 5''$; V to D $= \cdot 5''$; D to P.R. line $= 3''$; P.R. line to $W_p = \cdot 5''$; W_p to Block Coefficient line $= \cdot 5''$; Block Coefficient to E $= \cdot 5''$.